Magic, Probability and Quantum Mechanics

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This essay is about a puzzle of the kind raised by a magician with the intention of bamboozling his audience (possibly to make money!). I shall give an account of it here, but my interest in it is not in bamboozling the reader (or indeed making money). Some of you may have come across it before, but I wish give some observations of mine that seem relevant to the greater puzzle that is quantum mechanics.

First the magic bit. It concerns the trick where the magician has three cups laid upside down on a table and under one he places a coloured ball. Then he rapidly moves the cups around so as to confuse you, the punter, as to which cup the ball is actually under. Your task is to guess which cup contains the ball. So you take a guess and point to one. Then the magician promptly overturns one of the two cups you did not select and shows that the ball is not there. He then looks at you and invites you to change your mind and pick the remaining cup instead. So the question is: do you stick with the cup you originally chose or go for the cup the magician suggests? Your first thought might be that nothing has really changed so you may as well stick with your first choice, but you would be wrong.

The assumption that nothing has changed is false. Something most definitely has changed – and that is your information about the state of things. Previously you did not know for certain which of the cups the ball was under, now you know for sure it wasn't under the one overturned by the magician. If you think this makes no difference, you should realise that, had there actually been a ball under the overturned cup, you would know exactly where the ball was – and that the game would be over. Your new knowledge would have changed the game altogether. The fact that the ball wasn't under that cup seems less significant, but it still told you something – the question of which cup the ball is under is now confined to two cups, not three. This knowledge must improve your chances of getting the right answer.

All right then, the ball is under one of two cups. So you might think there is now an equal chance that the ball is under your chosen cup or the magician's suggested cup. So on that basis there is no reason to change your mind. Again, you would be wrong. To see why, lets look at the problem from the point of view of *probability*.

To begin with, when the ball is under one of three cups, but you don't know which, the probability that it is under any one of them is one in three, which is a 1/3 probability[1]. When you pick a cup, there is a 1/3 probability that you have the right cup. However there is a 2/3 probability that the ball is under one of the other two cups, taken collectively. (Taken individually, the probability that it is under each of the two cups is still 1/3.) When the magician overturns one of these cups, there are two possible outcomes: the cup contains the ball (in which case the game is over) or the cup does not contain the ball (and it is still game-on). If it is the second case, we know that originally the two cups collectively held the ball with 2/3 probability, but now we know that the probability is confined to just one cup – the one the magician did not turn over. So the unturned magician's cup now has the 2/3 probability of containing the ball. This is *twice* the probability (1/3) of your first chosen cup. So if you are smart, you will change your mind. The magician of course thinks you are dumb and will stick with your original choice, so he has twice the chance you have of winning. A knowledge of probability allows you to turn the odds in your favour!

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1 This is because the total probability must be unity (the ball must be *somewhere*!) and each cup has an equal probability of containing the ball, so that's a 1/3 probability for each.

What I like about this trick, and the observation I wish to make, is how probability seems to move from one cup to another with a change in your information about the system. This is not remarkable when you realise that probability is merely a mathematical expression of the state of your knowledge (or ignorance) of the system. So it is inevitable that probability changes when your knowledge changes. It's not as if in changing the probability you are actually moving the *ball* around between the cups, which would be absurd (wouldn't it?), but you are changing where you *think* the ball might be. I say *might* because you only get to *know* where the ball really is when you finally turn over the right cup and expose it[2].

This is where quantum mechanics come into mind. Quantum mechanics describes how fundamental (so-called 'quantum') particles like electrons and protons behave. It was realised very early on that a quantum particle is so small the very act of attempting to observe where it actually is makes it shift its position, which remains uncertain . From this the realisation grew that we cannot know *even in principle* precisely where such a particle is. All we can say is that the particle must exist *somewhere.* To keep the story short, let me just say that, *a*ccording to the theory of quantum mechanics the location can be described by a mathematical function called the *wavefunction* which defines the *probability* that the particle is in any given position.

There is a certain inevitability about all this. If we cannot say precisely where something is we can only state its position in terms of probability. The predictions arising from quantum mechanics are couched in probabilistic terms, unlike those of Newtonian mechanics, where predictions of location are truly precise. So it turns out that in quantum mechanics we do not get precise predictions of where quantum particles are, we can only predict the probability that it will be in a particular location. Experiments performed on quantum particles confirm this. A particle fired at a screen can hit it anywhere, but if many experiments of this kind are performed, it becomes apparent that it is more likely to strike the screen at some places than others – a kind of density pattern emerges – like shotgun pellets scattered over a target. It is a remarkable result, but one quantum theory predicts beautifully. However, in contrast to Newtonian theory, quantum mechanics cannot tell us which path the particle took between gun and target. In fact, according to the Path Integral theory of American Scientist Richard Feynman the particle seems to take all possible paths simultaneously! However, behind all this complexity one simple idea remains: quantum theory describes the motion of a particle in terms of the probability: firstly the probability of the particle being at the first position and secondly the probability of it later being at the second position.

This brings us back to the ball-and-cups system we discussed above. In that account it was noted that the shift in probability between cups does not mean that the ball is also physically shifted and that it was absurd to think so. But in fact, since we do not know where the ball actually is until it is finally revealed, we cannot be certain that the ball has not indeed moved! That we 'know' it has not is entirely based on our life experience of a world underpinned by Newtonian physics. But in the quantum world it cannot be a violation of the laws of physics if those laws cannot tell us for certain where the ball is anyway. All we have is the wavefunction that describes the system and must follow what the wavefunction does. If the wavefunction changes, probability changes, and that is what decides where the ball is most likely to be. It is a consistent description. In the quantum world it would seem that when the probability moves, the ball must move with it.

Once it is accepted that the precise location of a quantum particle is impossible even in principle, then the language of quantum mechanics is irreducibly bound to the principles of probability and probability is not bound by any law of physics save one – the particle must exist somewhere.

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2 Of course this presupposes the magician has not slyly stolen the ball by sleight of hand – which could happen with an unscrupulous magician bent on winning all the time!