Newtonian Gravity

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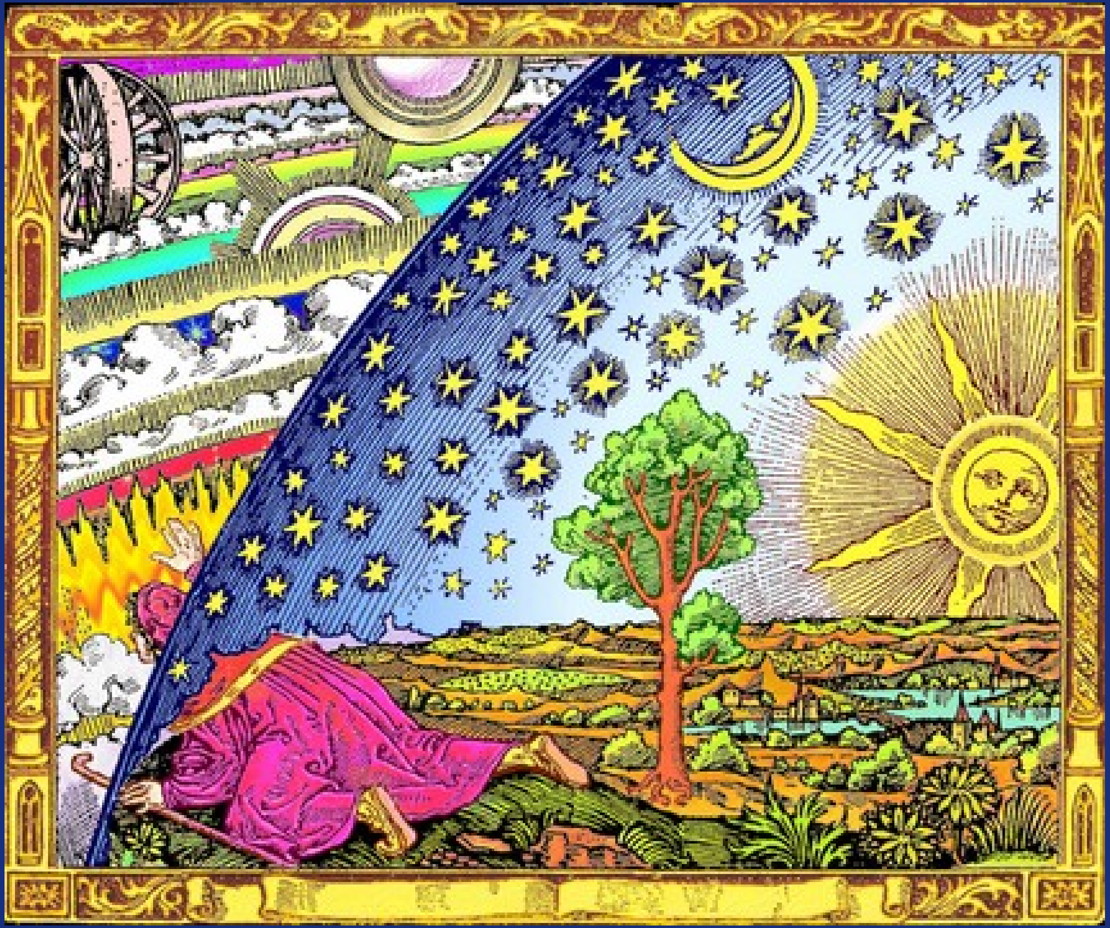
2021

Newton’s Impact

Camille Flammarion, L'Atmosphère: Météorologie Populaire (Paris, 1888), p. 163



Newton’s Impact

- I’d like to start with this picture, which shows something of the essence of the science that emerged from Newton. (Though the picture itself is relatively recent.)
- Here we have the world of familiar things, the stars, Sun, Moon, Earth, trees etc. and here is a philosopher (I presume) who has pierced the sky and discovered that the world is actually run by mechanisms and processes unseen by our normal perceptions.
- It is the role of science to uncover this hidden world and Newton was the first person to reveal what it was *really* like.

Introduction

- In this talk, I will introduce you to the theory of gravity, as it was first presented by Isaac Newton in his book Principia, which was written in 1687.
- Its development is one of the greatest scientific achievements of the last millennium and a testament to the genius of Newton.
- Despite being superseded by Einstein’s General Theory of Relativity in nineteen fifteen, it remains a remarkably successful theory and is still fully employed today.
- As an historical background I will introduce some of the predecessors of Newton who laid the groundwork for his theories. Newton claimed he stood on the shoulders of giants and these are undoubtedly some of the giants who influenced him.
- Next I will talk about Newton's famous and difficult book – the Principia, and provide an overview of the contents as are relevant to gravitation. Hopefully you will appreciate what a strange and powerful book it is and see it as a reflection of Newton’s exceptional mind.

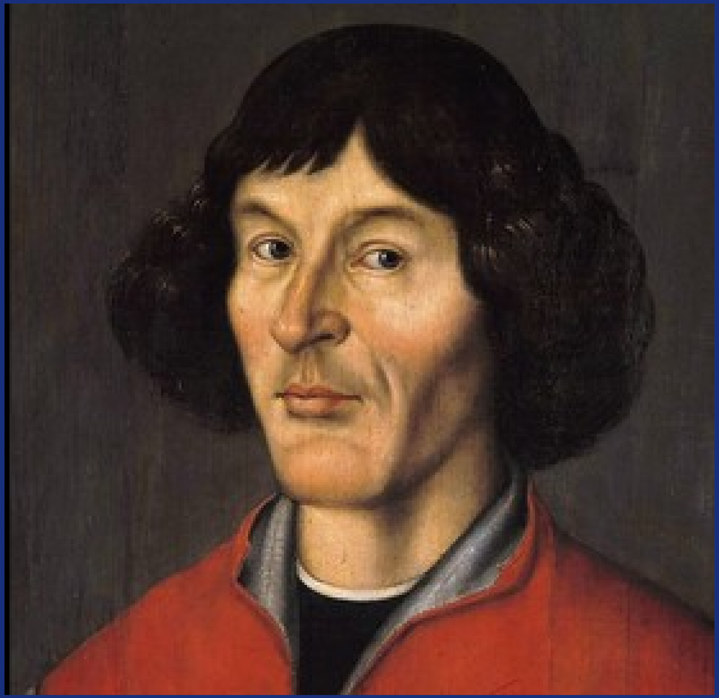
Introduction

- I will then follow this with some examples of how Newton applied his theory to phenomena beyond planetary motion and which had defied explanation before his time. In most cases his solutions were completely unexpected.
- Finally, I will mention the scientists who followed Newton and raised his theory of gravitation to the exact science it became. Some of these scientists were initially sceptical of Newton's work, but were ultimately compelled to accept it as correct.

Gravity in a Nutshell

- Gravity is the weakest natural force, yet it governs the universe - from the motions of planets, to the rise and fall of the tides, the formation of the stars, the dynamics of galaxies and the evolution of the universe.
- The weakness of gravity is apparent from comparing the electric force between two electrons with the gravitational force. The electric force is stronger by a factor of 4.17 times 10 to the power 42. If you prefer a conventional notation, I have written it here below. It is a colossal number!
- That’s why a small magnet (which represents an electromagnetic force) can pick up a paper clip despite the gravity of the whole Earth pulling against it.

Nicholaus Copernicus 1473-1543



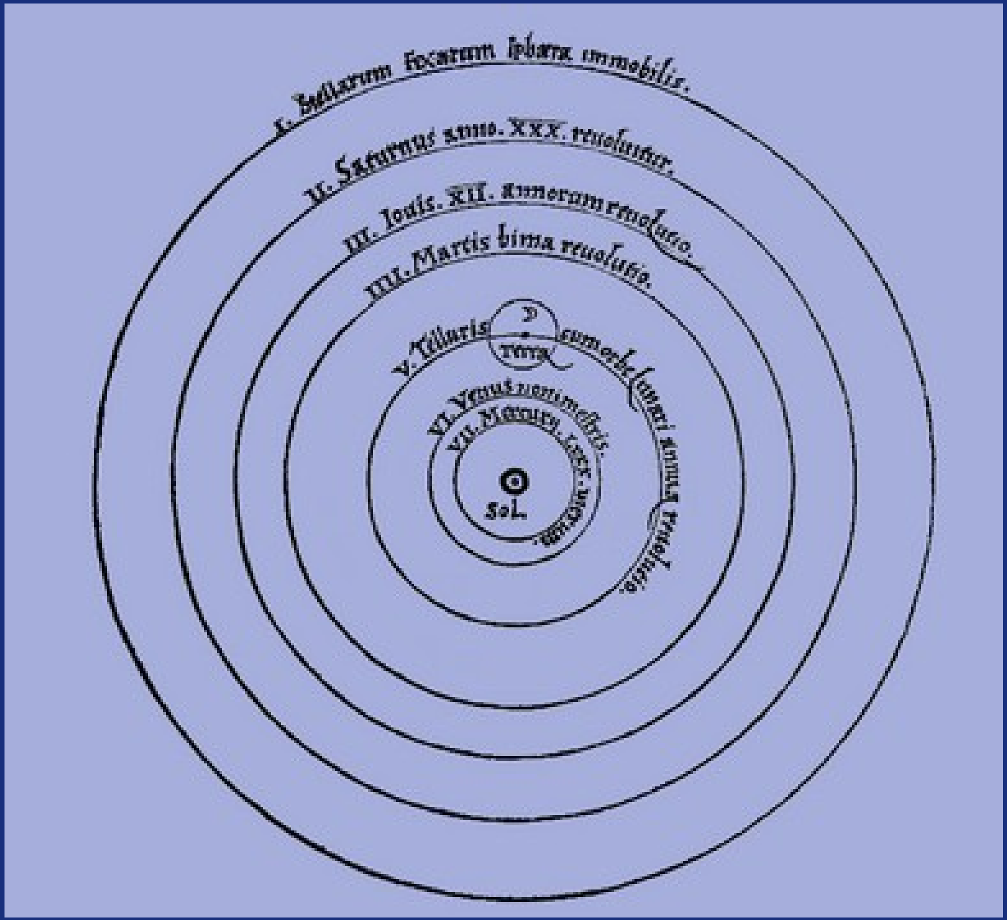
- Mathematician
- Astronomer
- Economist
- Physician
- Diplomat
- Scholar

Nicholaus Copernicus 1473-1543

- The first of Newton's predecessors I want to mention is Copernicus, who was clearly a man of great intellect, as revealed by this list of his talents, which extend far beyond astronomy alone.
- He was the first person of the modern era to propose that we inhabit a Sun centred world (or universe, as we would say today).
- Prior to Copernicus, the Ptolemaic model prevailed, in which the Earth was the centre of all things and the Sun and all the planets revolved around it.
- Until Copernicus had demonstrated the Sun centred model, a dynamical theory of the heavens involving gravity could never have been imagined.

The Helocentric Universe

De revolutionibus orbium coelestium



On the Revolutions of the Heavenly Spheres
N. Copernicus 1543

The Helocentric Universe

- This is Copernicus' model, with the Sun at the centre and all the planets revolving in circles about it. The one exception is the Moon, which revolves about the Earth.
- The planets were assumed to be fixed to transparent 'crystal spheres' that revealed successively higher levels of the universe out to the sphere of the fixed stars.
- This model offered a more natural explanation of the motions of the planets as seen from Earth, without deferents, epicycles and equants *etc.* the Ptolemeic model required
- Despite offering greater simplicity, Copernicus' model was, in fact, no more accurate than the Ptolemaic model.
- Copernicus' book: 'On the Revolutions of the Heavenly Spheres' was published the year he died (1543). So he escaped any possible sanction by the catholic church, which held to the Ptolemaic model as doctrine.
- Copernicus almost certainly believed in the correctness of his model, but his publisher inserted a caveat that it was merely a model to simplify the mathematics and not absolute truth..

Tycho Brahe 1546-1601

- Astronomer
- Stellar cartographer
- Destroyed the ‘Crystal Spheres’
- Anti-Copernican
- Duellist!

Tycho Brahe 1546-1601

- Tycho Brahe was an extraordinary man, an irascible duellist who was also the last great naked-eye astronomer. He built large instruments to measure the positions of the stars and planets more accurately than anyone had before.
- He measured the positions of comets and showed they moved between the ‘Crystal Spheres’ and thus demonstrating they were fictitious. He also observed a famous supernova in 1572 and proved it was a stellar rather than an atmospheric phenomenon, which overturned the prevailing belief.
- He opposed the Ptolemeic system, but rejected Copernicus’ solution. His own idea was that the Earth was at the centre of all, but the planets all orbited the Sun which, with the Moon, went around the Earth! Logically this is the same as Copernicus’ model, but lacks its simplicity.
- Tycho’s great contribution to astronomy was his accurate charting of the positions of the stars and planets. These were invaluable when his successor Kepler determined the true motion of the planets.

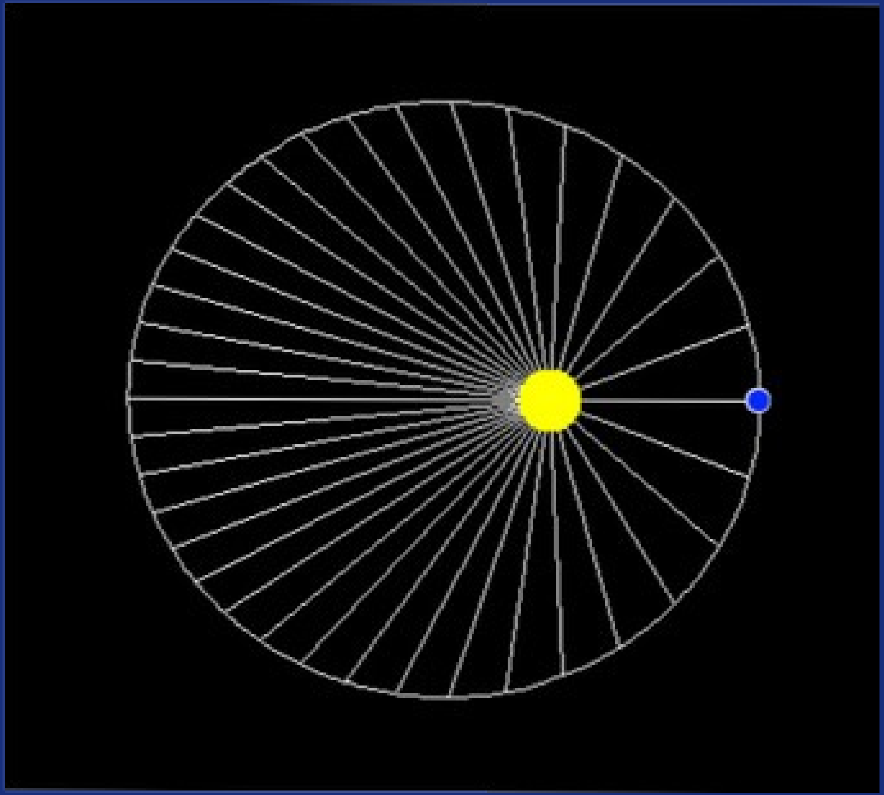
Johannes Kepler 1571-1630

- Assistant to Tycho Brahe
- Mathematician
- Astronomer
- Inventor
- Compiled the Rudolphine Tables

Johannes Kepler 1571-1630

- Kepler was a great mathematician, with a flair for thinking outside the box. Tycho initially hired him to bring order to his observational data prior to publication.
- A priority for Tycho was for Kepler to confirm his model of the solar system. But he died before Kepler could finish the work, which freed Kepler to try different approaches.
- Kepler found that none of the models of Ptolemy, Copernicus or indeed Tycho could account accurately for the positions of the planets.
- He therefore took a radical approach and in 1609 found a completely new model that gave accurate predictions of planetary positions. This model is encapsulated in Kepler’s three laws of planetary motion.

Kepler and Planetary Motion



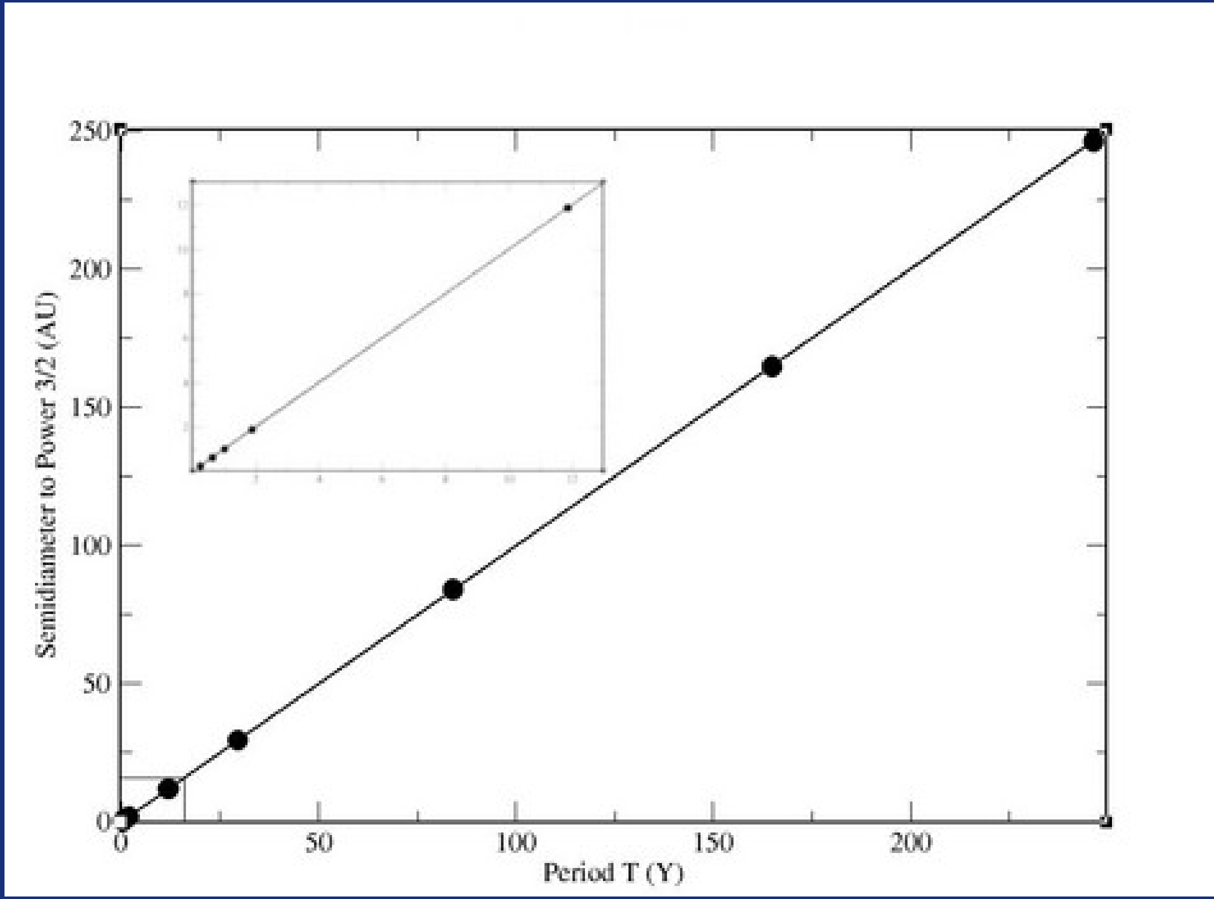
Kepler's three laws:

- Planets orbit in ellipses with Sun at one focus.
- Radius vector sweeps out equal areas in equal times.
- R^3/T^2=constant.

Kepler and Planetary Motion

- Kepler's first law is that the planets follow elliptical orbits with the Sun at one focus, as is shown in this diagram. (An ellipse generally has two focii, which makes one wonder what is the role of the empty focus!) The ellipse here is exaggerated; planetary orbits are very nearly circular.
- Kepler's second law says that the line drawn between the Sun and the planet (the radius vector) sweeps out equal areas in equal times. In the example here all the indicated positions are equally spaced in time, but the sectors of the ellipse enclosed between radius vectors are equal. This means the speed of the planet when close to the Sun is greater than when it is distant from the Sun.
- Kepler used the second law to compute the positions of planets at any given time with great accuracy, using a method now known as Kepler's equation, which is still in use today.
- Kepler's third law came much later (1619). It relates the mean distance of the planet from the Sun, R, to the period of the orbit, T. The ratio of R cubed over T squared is a constant.
- All Kepler's laws were discovered by a careful examination of the data. There was no physics involved and Kepler had no explanation for them.

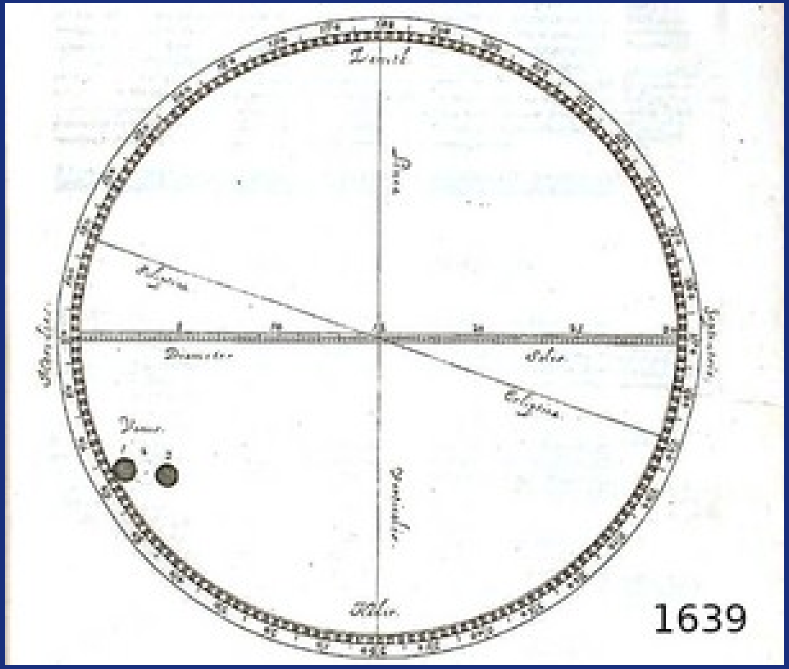
Kepler’s Third Law



Kepler’s Third Law

- Kepler’s third law is remarkably accurate. You can check this for yourself using data from Norton’s Star Atlas. This figure shows how it holds for all planets - as shown in the main figure. (The smaller window shows the same plot for the inner planets alone.)
- I have plotted the average radius of the planetary orbit (raised to the power 3/2) against the orbital period. This is mathematically the same as Kepler’s law.
- This result is a good indication of what is meant by the term ‘exact science’ - a sign that planetary motion can be determined precisely. Newton’s theory of gravity was to became the arch example of an exact science.

Jeremiah Horrocks 1618-1641



Transit of Venus

Jeremiah Horrocks 1618-1641

- The power of Kepler’s laws was well demonstrated by the work of Jeremiah Horrocks, who’s brilliant career as an astronomer was tragically short.
- He was a dedicated observer of planetary motion and charted the planets with his telescope.
- He sought to compute the movements of the planets precisely, but found that neither the Ptolemeic nor Copernican models was accurate enough. So he tried Kepler’s model and found precise agreement with his own observations.
- He later found, when he calculated the orbit of Venus, that it would transit across the face of the Sun on 24th November 1639 (by the Julian calendar). Kepler had missed this in his calculations and had predicted a near miss.
- Horrocks, located in Much Hoole, Lancashire, and his friend William Crabtree, located in Broughton, Manchester, were the only people on Earth to observe the transit, despite the handicap of the poor English weather on the day.
- Horrocks wrote an account of the transit in a monograph: *Venus in Sole Visa* (Venus on the Face of the Sun) and died shortly after in 1641, aged just 22.
- The monograph was not published until 28 years later - by astronomer Johannes Hevelius in the Netherlands. It created an intellectual storm.
- Horrocks’ importance is that he proved the essential accuracy of Kepler’s laws. This and other work by Horrocks – particularly on the motion of the Moon - was a considerable influence on Newton.

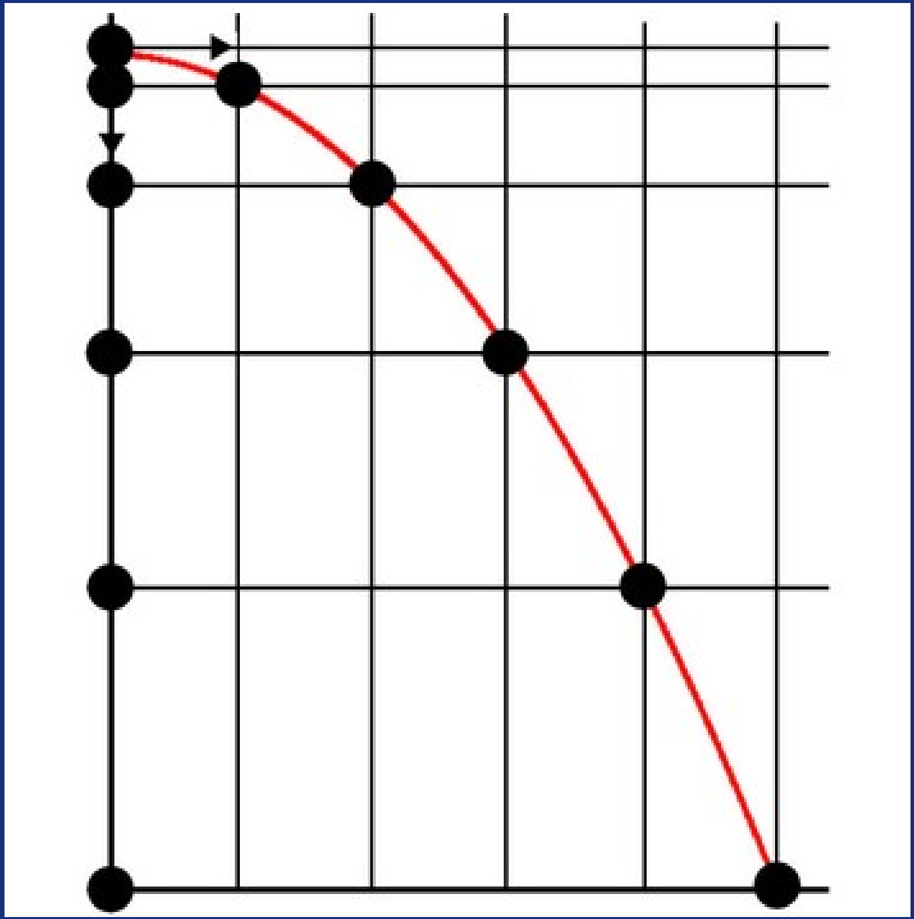
Galileo Galilei 1564-1642

- Astronomer
- Physicist
- Mathematician
- Experimentalist
- Engineer
- Inventor
- Copernican and Heretic

Galileo Galilei 1564-1642

- Galileo is arguably the first modern physicist, whose skills overlapped with Newton’s. A fierce Copernican, he fell into conflict with the catholic church, which resulted in him spending his final years under house arrest in Florence.
- As an astronomer, Galileo was famous for discovering the ‘Galilean’ Moons of Jupiter, the craters of the Moon and the phases of Venus – all achieved with a telescope of his own invention and manufacture.
- As a physicist he devised ingenious experiments to analyse the motion of bodies and made important advances in the theory of motion.
- He was the first to undertake a systematic analysis of motion and the nature of force. He made the distinction between uniform and accelerated motion and corrected false assumptions that abounded in the subject.
- For example, he showed that all falling objects accelerate at the same rate, in contrast to what was believed at the time. This is a profoundly important observation, particularly for modern physics.
- Galileo’s method of combining detailed experiment with mathematical invention anticipated the approach of Newton.

The Physics of Motion (Galileo)

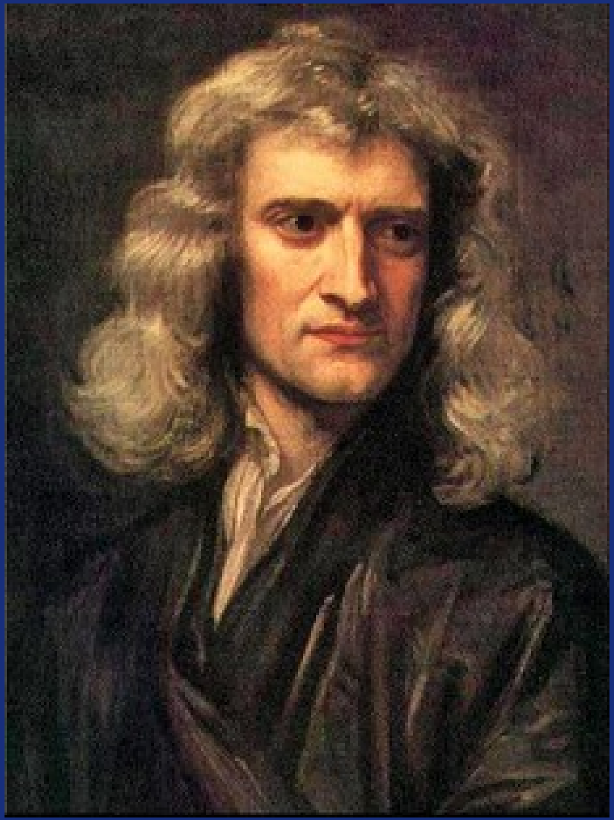


- Pendulum and time
- Motion on an inclined plane
- The persistence of motion
- Separate horizontal and vertical motion
- Derivation of projectile trajectories.

The Physics of Motion (Galileo)

- Galileo pioneered the use of pendulums to measure time accurately. He performed experiments with bodies rolling down an inclined plane, for which the motion, being slower, was more amenable to investigation.
- He understood that objects set in motion would in principle move forever and only come to a halt because of friction. This is the root of the principle of the conservation of momentum.
- He understood that force was not required to keep objects moving, but was required to accelerate them. These ideas were incorporated into Newton's equation of motion.
- He showed that the trajectory of a body moving though space could be described by combining the separate vertical and horizontal components of motion.
- In the figure, the vertical motion is accelerated and proportional to the time squared. The horizontal motion is uniform and proportional to time. These two motions combine to give the curved (parabolic) trajectory in two dimensions.

Isaac Newton 1642-1727

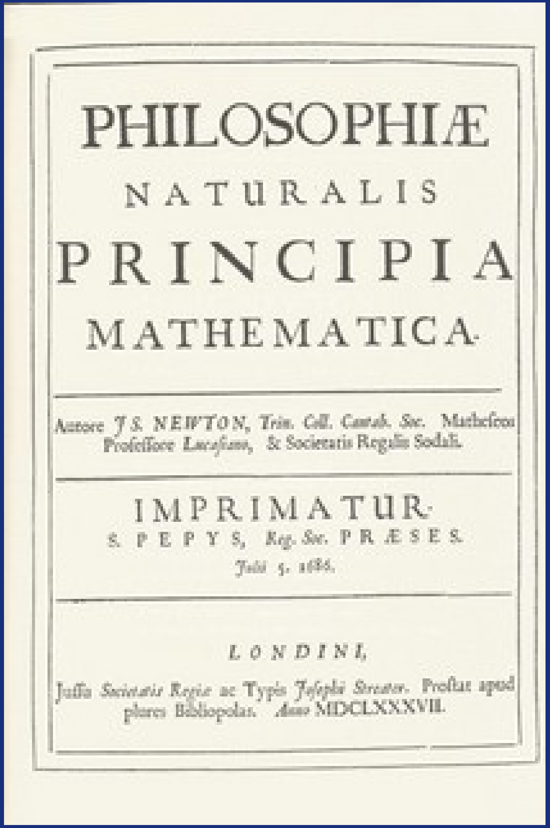


- Mathematician and Physicist
- Experimentalist
- Astronomer
- Inventor
- Alchemist
- Biblical scholar
- Master of the Royal Mint
- Member of Parliament
- President of the Royal Society
- Author of Philosophiae Naturalis Principia Mathematica (Principia)

Isaac Newton 1642-1727

- Newton is widely regarded as one of the greatest scientists of all time, with some justification. As the French mathematician Pierre-Simon Laplace noted: the universe can only be discovered once!
- A profoundly talented physicist, mathematician and experimentalist, he was also a noted observational astronomer and inventor (for example the Newtonian telescope).
- He spent as much time on alchemy and biblical studies as on anything else, which many consider a waste of his time, but Newton had broad intellectual interests and these were serious subjects of study in his day.
- He became the Master of the Royal Mint after Cambridge University and became very wealthy as a result. He was knighted for reforming the English coinage during the reign of William and Mary - not alas, for science.
- He was MP for Cambridge University for a short while, but his only known speech in Parliament was to ask for a window to be shut because of the draft! However, he did once defend the rights of Cambridge University against King James II, at some risk to himself.
- He was president of the Royal Society from 1703 to 1727.
- His greatest work was the book commonly called the Principia – the Mathematical Principles of Natural Philosophy, which started the scientific revolution.

Newton's Principia


PHILOSOPHIÆ
NATURALIS
PRINCIPIA
MATHEMATICA.

Autore J S. NEWTON, Trin. Coll. Cantab. Soc. Matheseos Professore Lucasiano, & Societatis Regalis Sodali.

IMPRIMATUR.
S. PEPYS, Reg. Soc. PRÆSES.
Julii 5. 1686.

LONDINI,
Jussu Societatis Regiæ ac Typis Josephi Streater. Prostat apud plures Bibliopolas. Anno MDCLXXXVII.

Principia's Title Page

- Published by the Royal Society 1687.
- The founding text of modern physics
- Cast entirely in geometry and written in latin.
- No equations appear in the text.
- A notoriously 'difficult' text to understand!

Newton’s Principia

- Newton’s Principia is a profoundly important text. It marked the final end of the Aristotelian world view and the birth of modern science. It revolutionised our understanding of the universe.
- It was formally published by the Royal Society, which is why its president Samuel Pepys appears on the frontispiece. But in fact the credit is almost entirely due to Edmond Halley.
- The book set out the laws that govern the dynamics of the universe (or ‘world’ as it was described then) and established universal gravity as the primary causal agent in the universe.
- It is cast entirely in terms of geometrical arguments and uses no equations or related analytical tools such as calculus. In consequence it is a very difficult text to understand.
- It has been widely debated why he did this, but it’s possible he wanted to make his mathematical arguments unassailable, and by using classical geometry he succeeded completely in this aim. Unlike his physics, nobody challenged his mathematics.
- The Principia set the agenda for the next 200 years of progress in physics, and to a significant extent, it still does.

Newton’s Laws of Motion

Newton’s three laws are the foundation of modern physics:

1) All bodies travel with constant velocity, or remain at rest, unless acted upon by a force.

2) The applied force equals the rate of change of momentum in the direction of the force.

3) Action and reaction are equal and opposite.

Newton’s Laws of Motion

Newton’s second law is often summarised in the equation: force equals mass times acceleration, but Newton did not express it like this in Principia. He merely stated that the acceleration was proportional to force and inversely proportional to the mass.

$$F = ma$$

Today it is hard to see how revolutionary Newton’s laws were, so it’s important to point out a few things.

Newton’s Laws of Motion

- Newton’s laws were a synthesis of the ideas of many people besides himself. However, these were vague and half-formed until he put them into the right framework.
- Nobody until Newton (not even Galileo) had a sure idea what a force was until he defined it correctly in his first law.
- Newton properly defined what momentum is. Previously it was a concept not widely used or understood.
- Newton expanded the concept of acceleration. He was the first to realise that something moving in a circle with a constant speed was nevertheless undergoing an acceleration.
- Before Newton, the third law was not properly understood - and a viable system of dynamics is not possible without it.
- In Newton’s day it was widely believed that there was a centrifugal force that pulled a body away from a centre of rotation. Newton showed there was no such thing. He proposed instead a centripetal force that pulls a body towards the centre, without which it would fly off along a tangent.

Newton's Law of Gravity

The gravitational force F_{12} between two masses m_1 and m_2 at a distance r_{12} apart is given by the (now famous) inverse square law:

$$F_{12} = -\frac{G m_1 m_2}{r_{12}^2}$$

Newton's Law of Gravity

- Newton's law of gravity states: the gravitational force F_{12} between two masses m_1 and m_2 separated by a distance r_{12} is given by the shown inverse square law.
- The minus sign on the right indicates it is an attractive force.
- The symbol G represents the universal gravitational constant. Newton never referred to this constant (though he was surely aware of it).
- The value of G remained unknown until Henry Cavendish determined it experimentally some 70 years later.
- This equation does not appear in the Principia. Instead Newton simply stated that the force was proportional to the product of the masses and inversely proportional to the square of the distance between them.
- He expressed the law in this way because his overall strategy was to represent the proportionalities in geometric terms and use geometry to show his deductions must be true. This approach is alien to modern mathematicians!

Newton’s Law of Gravity

- Other ‘discoverers’ of the inverse square law were: Robert Hooke, Christopher Wren, Edmund Halley, Christian Huygens and the French priest Ismael Bullialdus. However, if their work is closely examined it is clear their understanding is not based on sound physics. Nevertheless Newton acknowledged their discovery because he needed allies in making his case to the world.
- Hooke, Wren and especially Halley were the reason Newton wrote Principia in the first place. Halley visited him in Cambridge and repeated a discussion he’d had with Hooke and Wren on planetary motion and this re-awoke Newton’s interest in the subject, which he’d put aside years before.

Derivation: Inverse Square Law

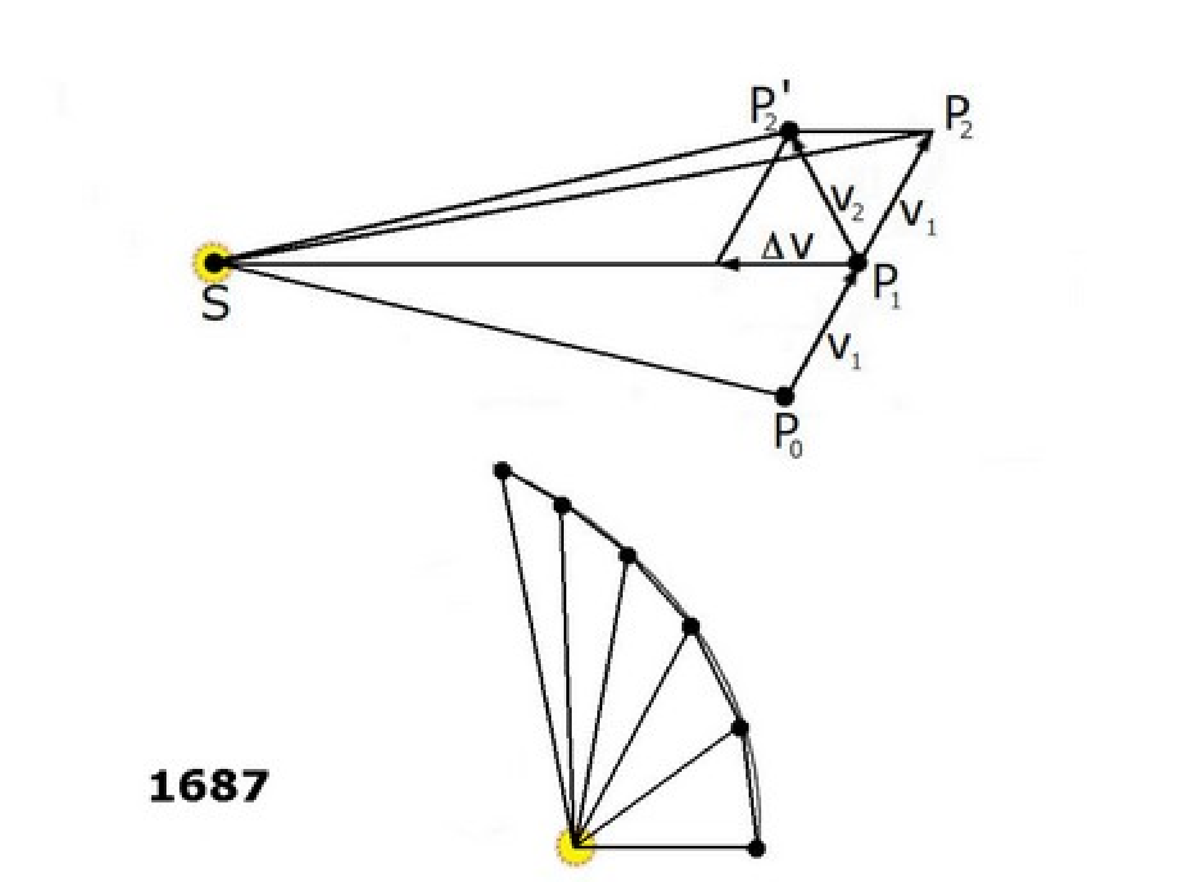
$$v = \frac{2\pi R}{T} \qquad T^2 = C R^3$$

$$F = \frac{m v^2}{R} = \frac{4\pi^2 m R}{T^2} = \frac{4\pi^2 m}{C R^2}$$

Derivation: Inverse Square Law

- The derivation of the inverse square law is very simple if the planet orbits in a circle around the Sun.
- Equation, top left: first we calculate the velocity of the planet from the circumference of the circle: 2 times pi times R divided by the period of the orbit T.
- Equation, top right: next we write Kepler's third law in this form, where C is a constant, the value of which we do not need to know here.
- Equation, bottom left: we have Newton's equation that shows the force F necessary to hold a revolving body in a circular orbit is the mass times the velocity squared, divided by the orbital radius.
- Now substitute for the velocity using our first equation – this gives the middle relation. Now replace T squared using Kepler's third law.
- The final result clearly shows the force is proportional to the reciprocal of the orbital radius squared.
- That was easy! However, proving the inverse square law is consistent with an elliptical orbit is a great deal more difficult. Nobody could do this before Newton.

Newton & Planetary Motion



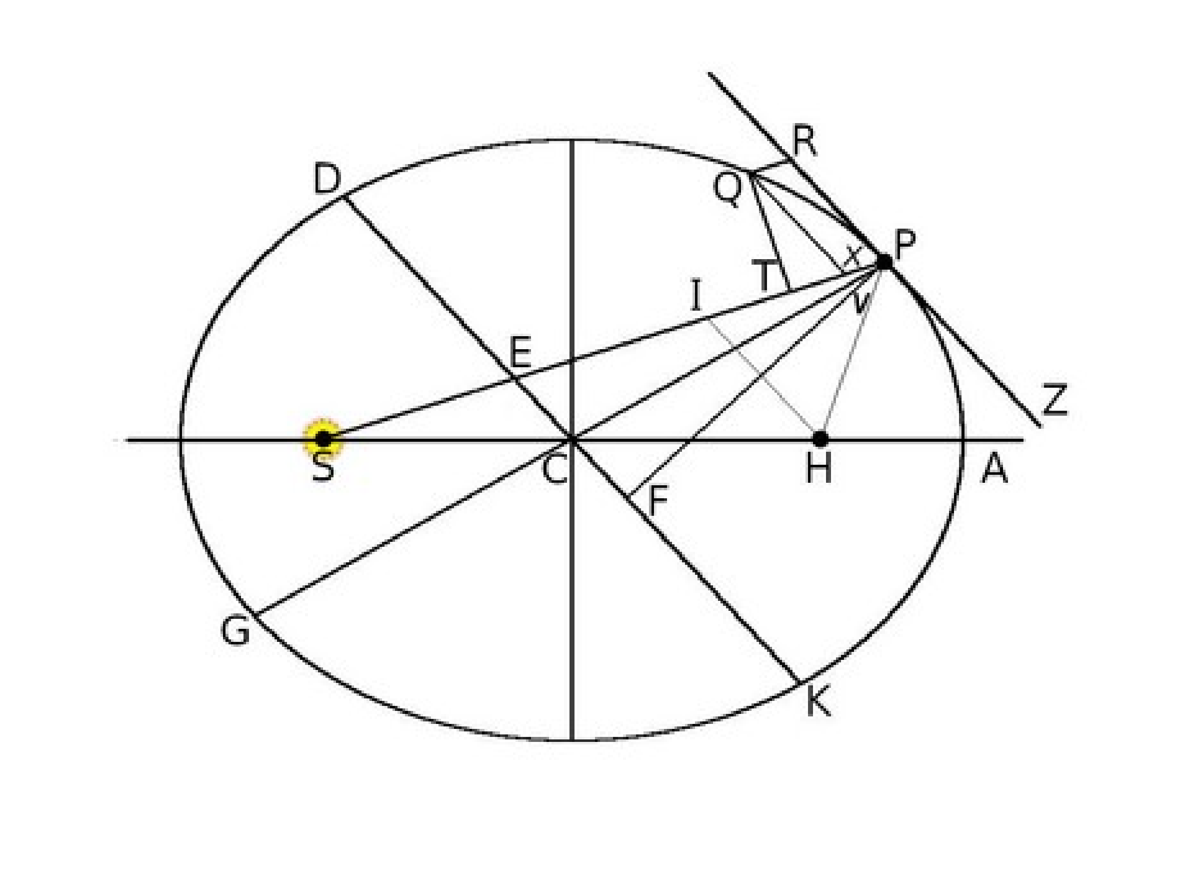
Newton & Planetary Motion

- This is Newton's proof of Kepler's second law concerning the area swept out by the planet's radius vector. It is astonishingly direct and simple.
- In the top figure we have the Sun at S and the planet is at position P_0. Ignoring gravity, we assume the planet travels with the velocity V_1.
- After a fixed interval of time it is at position P_1 and after another (equal) interval, it is at P_2 (by Newton's first law).
- Draw the radius vectors from S to the planet positions. Newton says the triangles S, P_0, P_1 and S, P_1, P_2 have equal areas. (Try and work out why!)
- Now, Newton says that if the planet at P_1 is struck instantaneously by a hammer blow in the direction of the Sun at S, there is an immediate change in velocity (δV) in that direction, which combines with the original velocity V_1 to send the planet to the position P_2', in the same time it would have gone to P_2. This follows from the construction of a parallelogram of the velocities V_1 and δV that combines them into the new velocity V_2.

Newton & Planetary Motion

- Newton now shows that that triangles S, P_1, P_2 and S, P_1, P_2’ have the same area. So triangle S, P_0, P_1 and S, P_1, P_2’ must also have the same area!
- Using this kind of argument, an approximation to an orbit can be created consisting of a polygon (bottom figure) in which the positions of the planet at the end of equal time intervals are obtained in this staccato manner. The areas of all the triangles between radius vectors must be identical.
- Splitting each triangle into sub-sections of equal time gives a better polygonal approximation to the true curve which still retains the equality of the enclosed areas. Continuing this process *ad infinitum*, the ultimate polygon and the orbital curve become indistinguishable and the equality of the areas is preserved, which proves Kepler’s second law.

Newton & Planetary Motion



Newton & Planetary Motion

- This is Newton's proof that a planet revolving around the Sun in an ellipse, with the Sun at one focus, must necessarily be obeying an inverse square law of gravity. This figure is arguably the most famous in the Principia and one of the most difficult to understand. I will attempt to explain the physics of what Newton is doing in a later slide.
- The full proof is difficult for several reasons:

  - The first is that it requires a deep knowledge of classical Greek geometry, principally the works of Apollonius of Perga on conic sections and advanced Euclidean geometry.

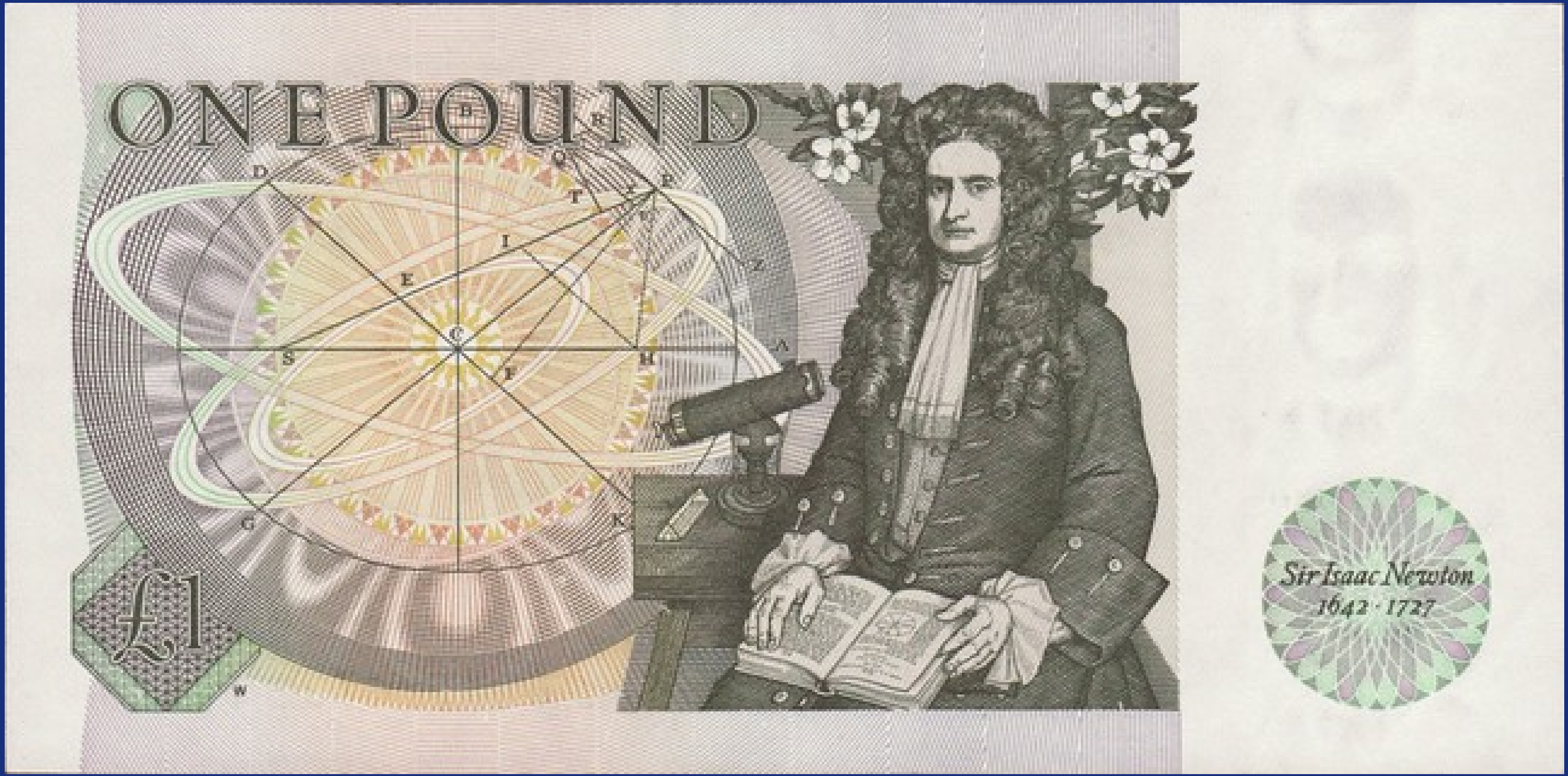
  - Secondly, Newton's technique of 'vanishing geometric ratios', which he invented, was entirely new and unique to this work. Very few people were (or are today) aware of it.

  - Thirdly, the principles of physics he applied were unknown in his day. Even today, they seem applied in a very unfamiliar way from what we're used to.

Newton & Planetary Motion

- Referring the diagram: the Sun is at one focus of the ellipse, at S. The empty focus is at H. The planet is at P and the line SP is the radius vector. The line RZ is a tangent to the ellipse at P and represents the direction of motion of the planet at P. Line PG is a diagonal through the centre at C and the line DK is another diagonal through C that is parallel to the tangent at P. Lines PG and DK are called *conjugate diagonals*. With this information, see if you can solve it on your own!
- While the brave do this, the rest of us will look at something more easily understood.

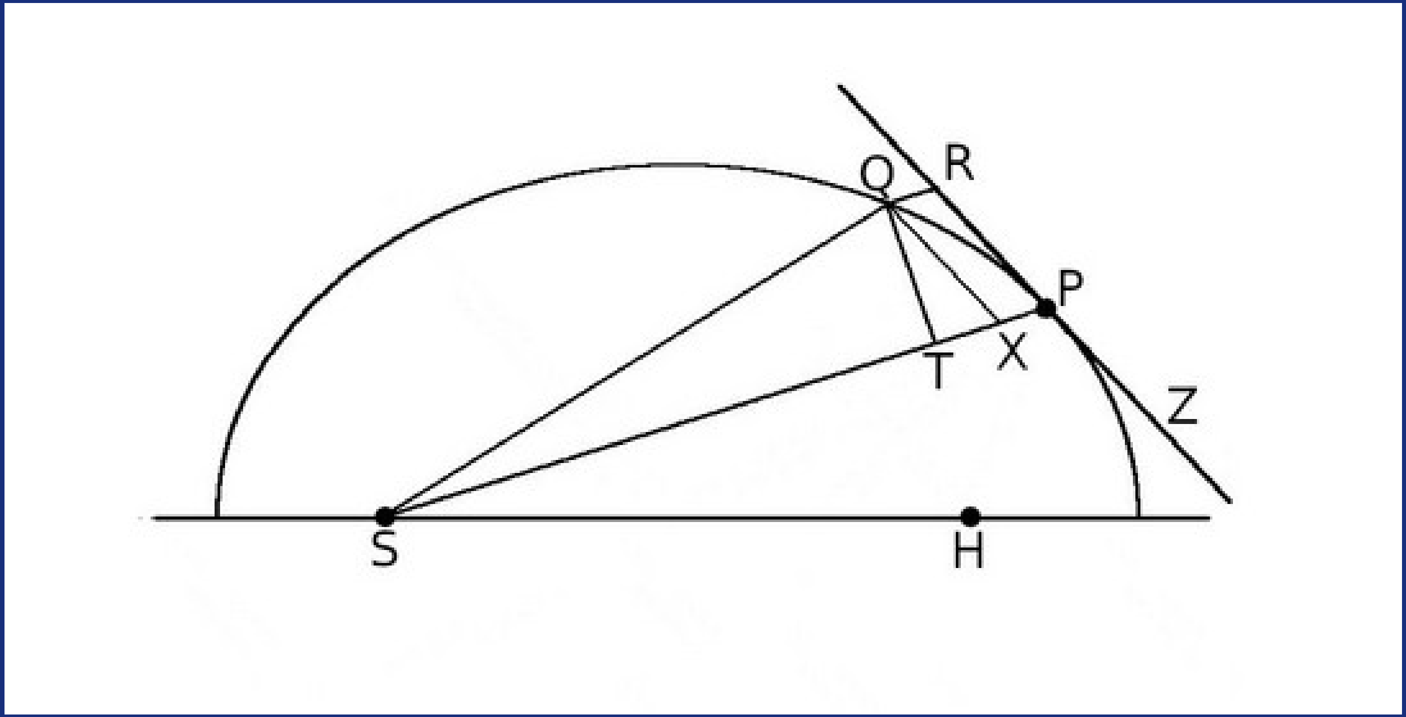
Newton and the Pound Note



Newton and the Pound Note

- I am sure you will recognise this as the reverse side of the old, one pound note, which celebrates Newton's achievements.
- Newton himself appears here with an open copy of the Principia in his lap. It is open at the page of the inverse square law proof we have just seen.
- On the table beside, is his original Newtonian reflecting telescope. He made this entirely by himself, including the grinding of the optical elements, without the assistance of any craftsman.
- Also on the desk is the prism Newton used to investigate the colour composition of light. This subject was his first published scientific study and it drew so much flak that he withdrew from publishing for nearly 20 years!
- Finally, in the background and almost hidden, is another rendering of Newton's inverse square law proof, complete in every detail, but with one glaring mistake by the note's graphic designer: the Sun is not supposed to be at the centre of the ellipse, but at the focus S!
- Nevertheless it is a handsome homage to the great man.

Newton & Planetary Motion



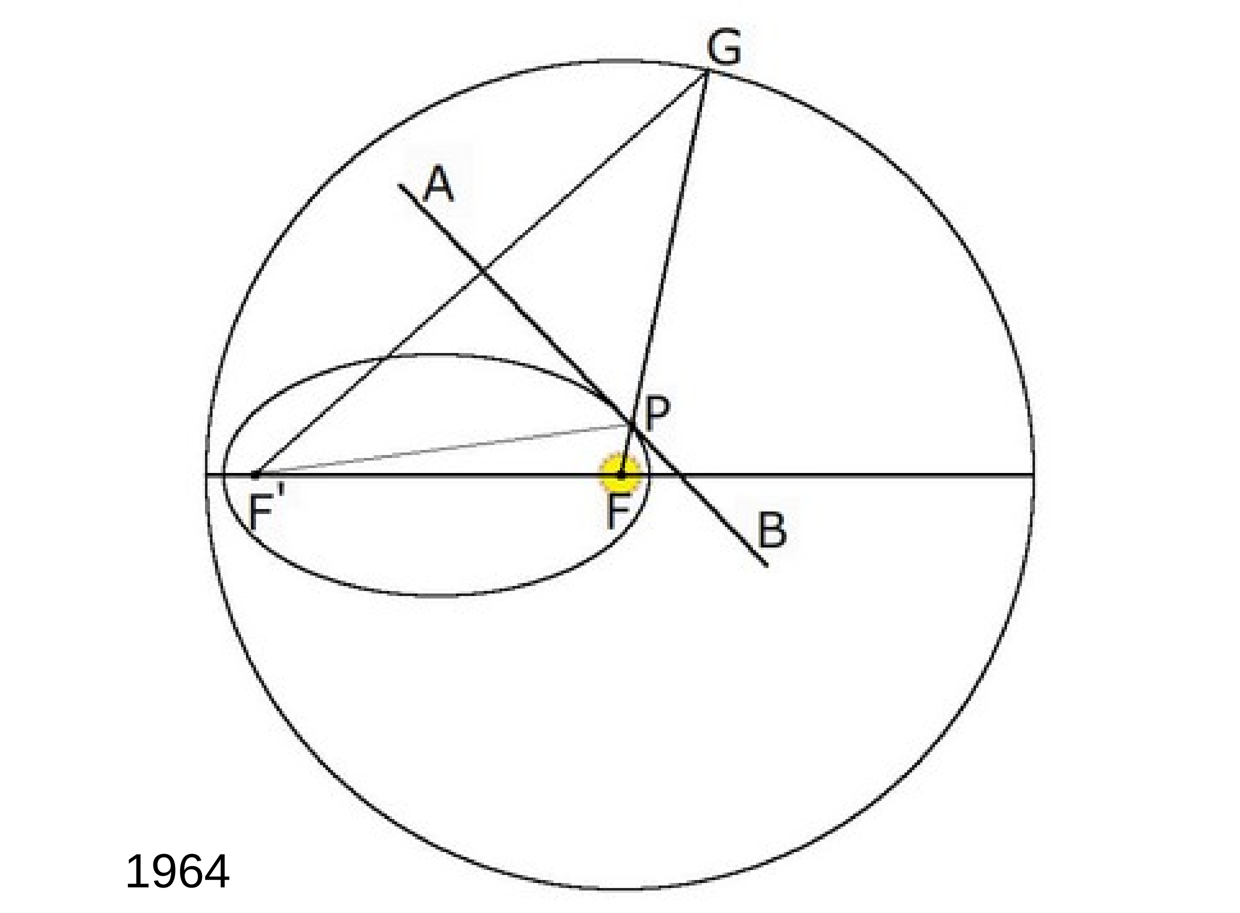
Newton & Planetary Motion

- Back now to Newton's proof. This is a simplified diagram that lays aside the classical geometry and focuses instead on the physics.
- Once again we have the Sun at S, the planet at P, with SP the radius vector. The line RZ is the tangent at P.
- Newton supposes that in a finite time the planet moves from P to Q. He then draws the line QR which is parallel to SP. Newton says the distance RQ is the distance the planet moves in the direction of the Sun (defined by the line S,P) as it moves from P to Q.
- He constructs another radius vector SQ and drops a line QT as a perpendicular to SP and finally draws a line QX parallel to the tangent at P. (The quadrilateral QRPX is then a parallelogram.)
- Now, the area of the triangle SQX with perpendicular height QT is taken as a rough approximation for the sector of the ellipse between radius vectors SP and SQ. But note, if we wind Q back along the curve towards P this approximation becomes better and better. We also know from Kepler's second law that the area of the sector is a measure of the time.

Newton & Planetary Motion

- Newton then defines a ratio of the distance QR (which equals XP) divided by the **square** of the area of the triangle SQX (which is the time squared) and says this ratio approximates to the acceleration of the planet towards the Sun at P.
- Newton then shows geometrically that as the point Q winds back towards P this ratio becomes a constant divided by the square of the distance SP and thus proves the inverse square law.

Planetary Motion by Feynman

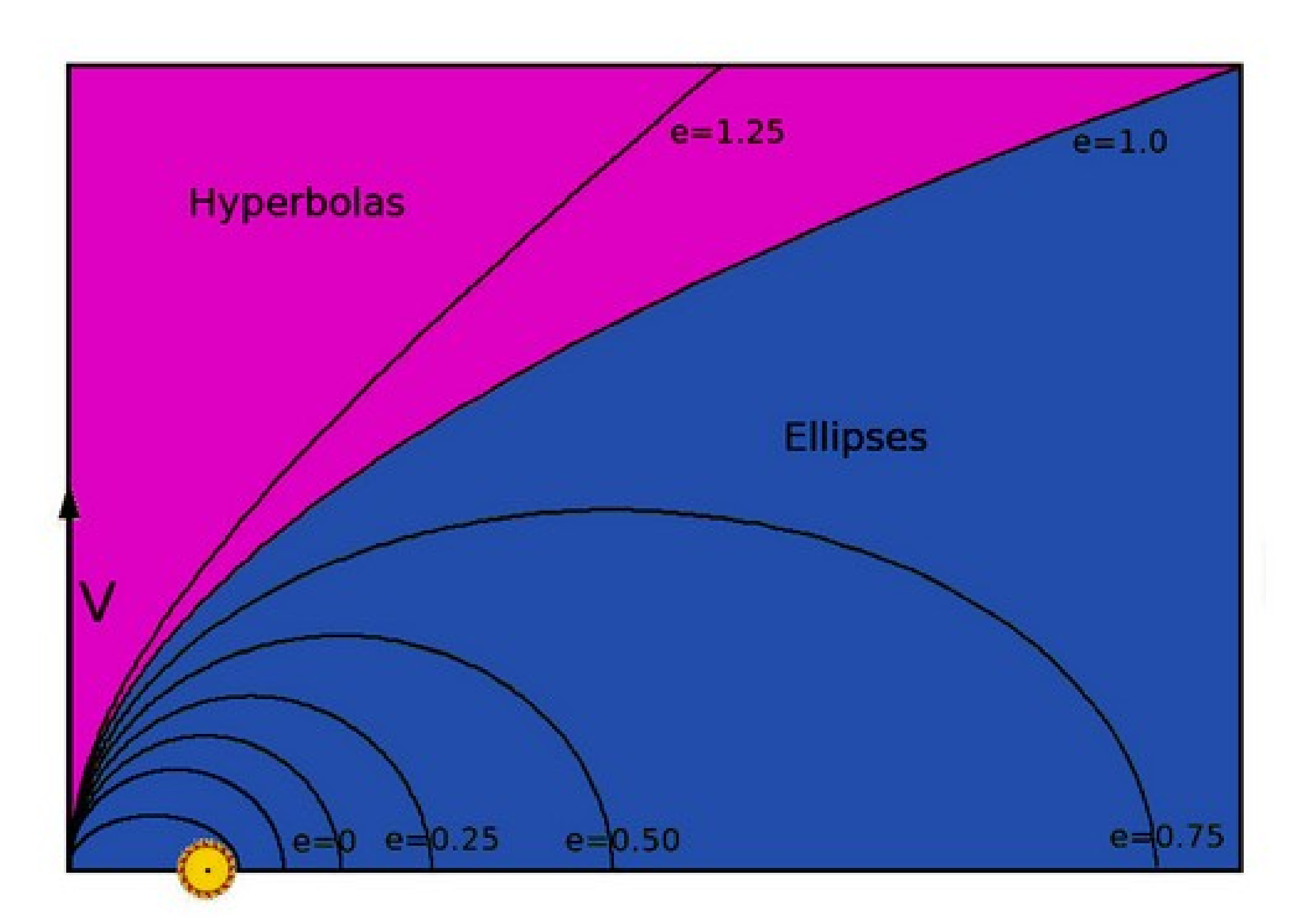


'Feynman's Lost Lecture'
D.L. Goodstein

Planetary Motion by Feynman

- This diagram is an alternative proof of the inverse square law by the American physicist Richard Feynman.
- Apparently he wanted to show his students Newton's derivation, thinking it would be easy to follow. But he soon discovered it was exceptionally difficult. But, being a genius himself he devised his own proof.
- I will spare you the details. These can be found in the book 'Feynman's Lost Lecture,' by D.L. Goodstein. I will however, mention two essential points.
- Firstly, the proof consists mostly of the classical Greek theorem that explains how to construct the tangent to the ellipse at point P. (The main point is that triangle F'PG is isocelles (two equal sides.)
- Secondly, Feynman found that, if the motion of the planet is governed by the inverse square law, the length of the line F'G (which is the base of the isocelles triangle), is directly proportional to the velocity of the planet at P.
- In other words this diagram lays out the relationships between all the principal elements of the dynamics – the planet's position, velocity and acceleration, which therefore must all be consistent with the inverse square law. This completes the proof.
- What is intriguing about this theorem is that it assigns a dynamical role to the second focus of the ellipse. The planet's position revolves in an ellipse around the first focus, while its velocity revolves in a circle around the second focus!

The Motion of Comets



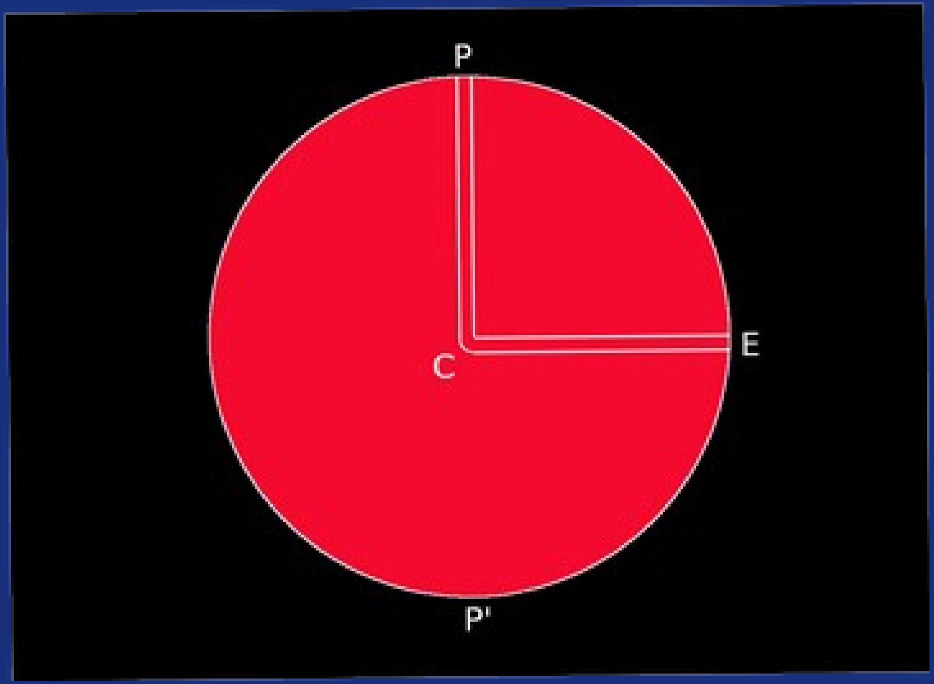
The Motion of Comets

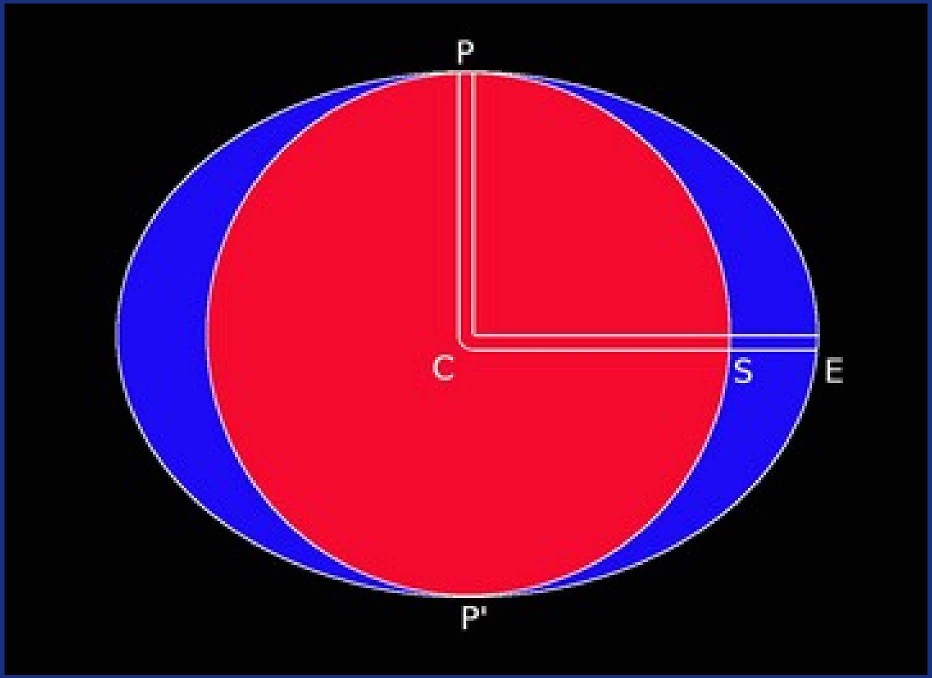
- Newton went on to prove that the parabola and the hyperbola were also consistent with the inverse square law of gravity. Note he did not do it the other way round, which is assume the inverse square law and derive the orbits that followed from that. Why? Because he wanted to show the inverse square law was a necessary consequence of observed planetary motion. He was trying to prove the law of gravity itself, not just assume it, which he thought was a questionable approach.
- Newton thus proved that all conic sections obey the inverse square law. So he now had a complete handle on the comets which, he realised, could have any of these orbits.
- The diagram shows the whole family of possible orbits. We have the Sun at the bottom left and a horizontal axis running through. At some point left of the Sun we draw a perpendicular to the axis. Now we consider what kind of orbits result from projecting a body along this perpendicular line at different speeds.
- For low speeds we get a family of ellipses, for which the Sun is in the right hand focus. The lowest velocities produce the most eccentric ellipses and they become more circular as the initial speed increases. Eventually the orbit becomes a circle and both focii merge into the circle centre.

The Motion of Comets

- Greater increases of speed now generate a second family of ellipses, in which the Sun is in the left hand focus. Higher initial speeds now produce ellipses of increasing eccentricity, until we arrive at a point where the ellipse becomes a parabola. At this point the orbit becomes open. The body will follow this path only once and not repeatedly as before.
- Even greater initial speeds produce hyperbolic orbits, which are also open.
- The difference between these orbits can be given in terms of energy. Bound orbits (ellipses, circles) have negative energy. Hyperbolic orbits have positive energy and parabolas have zero energy.
- The orbits of comets reflect their origins. Hyperbolic orbits imply an interstellar origin. Parabolic orbits imply an origin in the Kuiper belt or perhaps the Oort cloud. Ellipses imply an origin within the solar system. Alternatively they may be from farther out, but have undergone a prior interaction with a planet like Jupiter. Most comets have orbits that are close to parabolic.

Universal Gravity



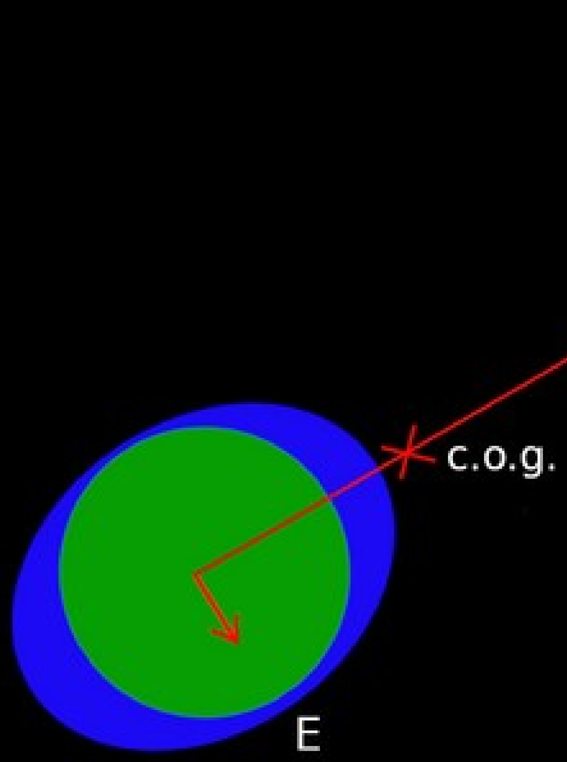


The Shape of the Earth

Universal Gravity

- Having defined gravity, Newton went on to explore its consequences in other contexts. Here he determined the shape of the Earth. He argued that the spinning Earth could not be an exact sphere, but should bulge outwards at the equator.
- With a stationary spherical world (left) wells drilled to the centre of the Earth from both the North pole and from the equator, should fill with water up to the Earth's surface identically. The liquid in each well is therefore gravitationally balanced.
- With an Earth that rotates about its polar axis however (right), the equatorial well must overflow, because the liquid in the well has angular momentum and without an additional centripetal force the water in the two wells is no longer gravitationally balanced. A head of water in a column above the equator provides the necessary downward force.
- The height of the additional column of water can be calculated using Newton's laws. This height is the degree to which the Earth must deviate from a sphere to achieve a stable surface.
- Newton used this idea to calculate the shapes of the Earth, Jupiter and Saturn, to reasonable accuracy (given that he didn't know what their physical compositions were.)

Theory of Tides

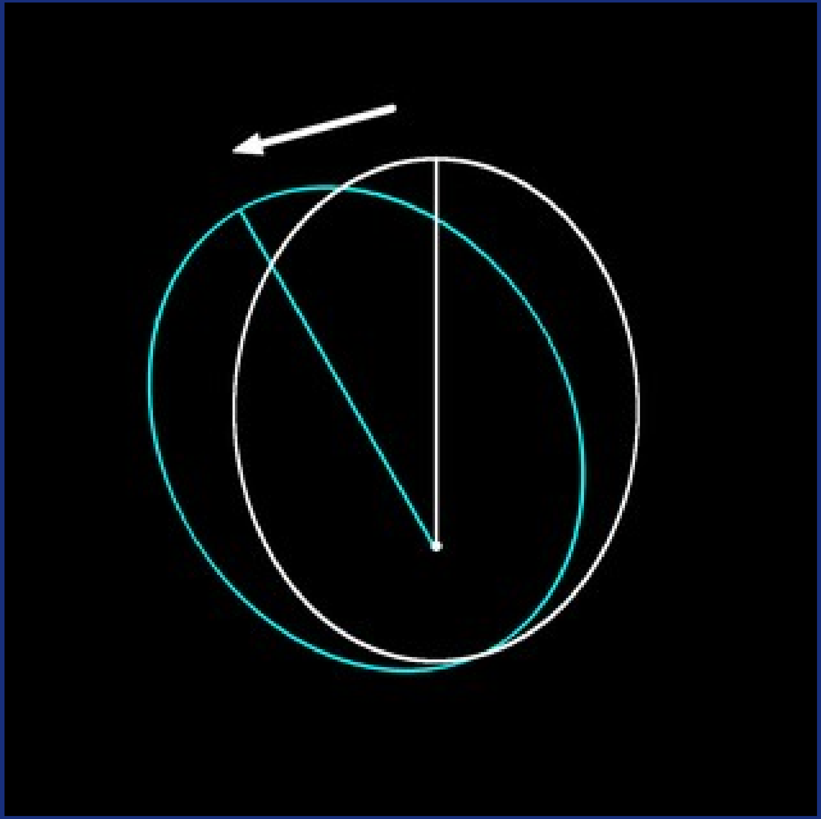


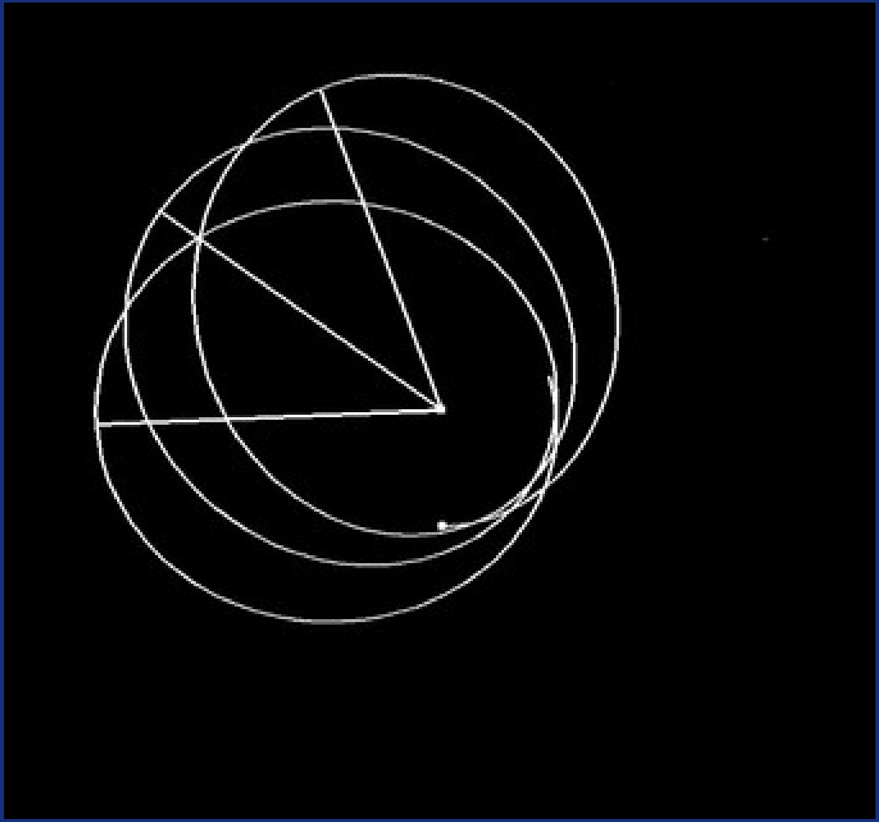
The Sun also exerts a tidal influence.

Theory of Tides

- Newton developed a theory of tides based on a combination of gravity and dynamics. He thereby provided an explanation of why there are two high tides in each day.
- The Moon above the Earth pulls more strongly on the Earth's near side than on the far side. This induces the water to 'bulge' out towards the Moon. The bulge is only about ten metres on account of the Moon's great distance.
- On the opposite side of the Earth from the Moon, the water again rises up in a bulge because the Earth-Moon system is rotating about the common centre of gravity. The Earth thus turns in a circle, while the water endeavours to travel along a straight line. In effect the water rises up because the Earth's surface is accelerating away from it.
- The tides on opposite sides of the Earth are about the same height.
- The tides are rotated slightly ahead of the Earth-Moon axis because the Earth rotates faster than the Moon orbits the Earth and this carries the bulge forwards.
- The Sun also exerts a tidal influence, which is ~45% that of the Moon's. It is not necessarily 'in phase' with Moon's tides and so can work with or against it to modify the tide's range.

The Motion of the Moon



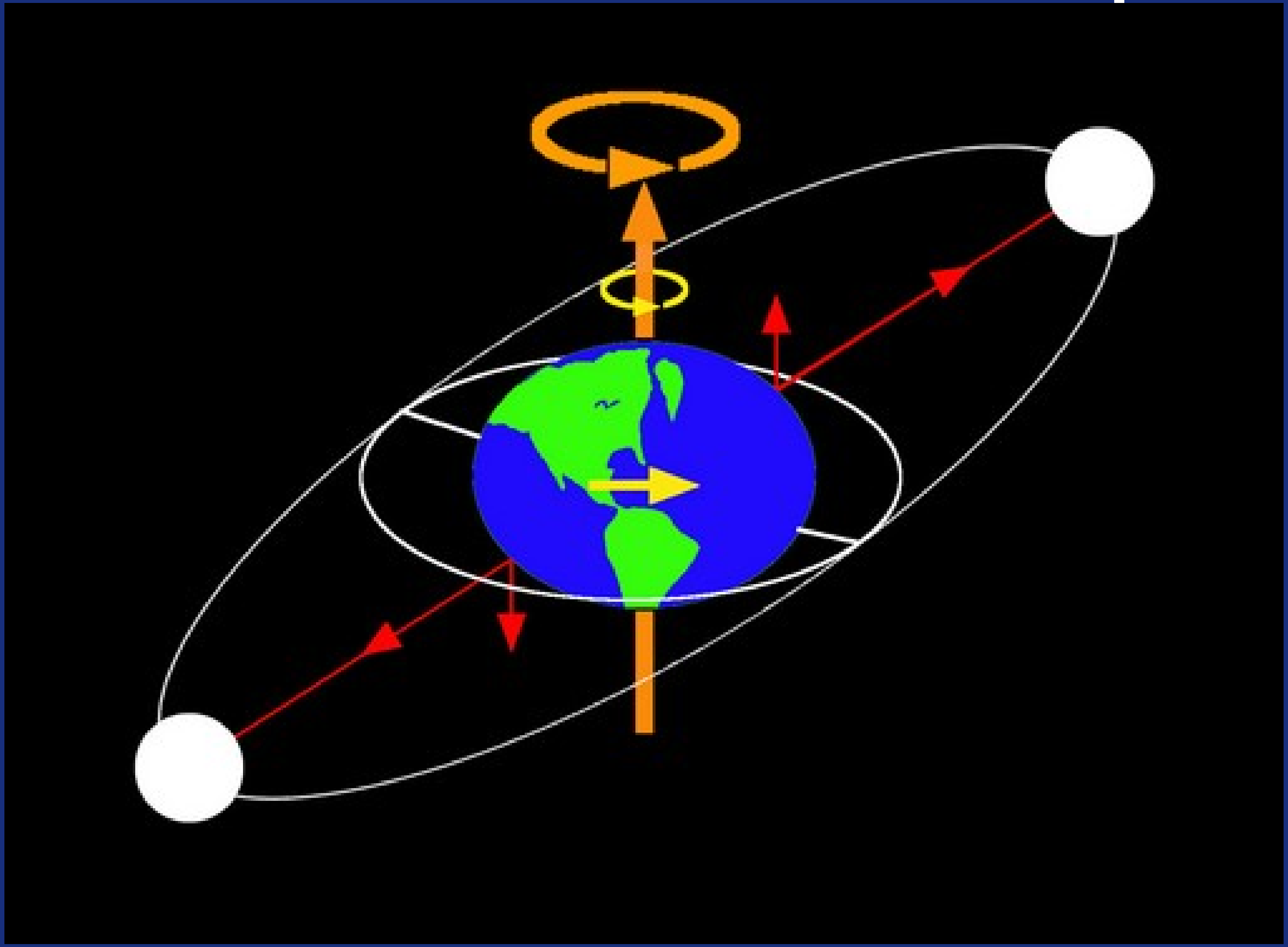


Precession of Apsides

The Motion of the Moon

- Newton's biggest challenge was to explain the motion of the Moon. When he addressed the problem he found that it was intractable. (Today it is known that the problem has no exact mathematical solution.) But he was able to make progress nevertheless.
- The problem involves the interaction of three bodies: the Moon, the Sun and the Earth, though the Sun's influence on the Moon is smaller than Earth's. Newton invented the method known as perturbation theory, in which the Sun acts as a small perturbation of the Moon's orbit around the Earth.
- He deduced that for a near circular orbit like the Moon's, there would be a precession of the Moon's apsides (which define the longest diameter of the Moon's elliptical orbit, see the Figure, left panel). In other words the elliptical orbit of the Moon would itself turn around the Earth. As the Moon follows its rotating elliptical orbit the result is a kind of 'spirograph' orbit (in the Figure, right panel).
- Unfortunately Newton's calculation of the precession rate was wrong by a factor of 2. (He later obtained the correct value but it did not appear in print during his lifetime.)
- French mathematician Clairaut (1749), proceeded in a similar vein, but to a higher level of approximation, and calculated the correct value. So Newton's method was vindicated.
- That Newton worked hard on the problem of the Moon is revealed in his quote to Edmond Halley: 'The motion of the Moon makes my head ache … I will think on it no more!'

The Precession of the Equinoxes



The Precession of the Equinoxes

- One of the most surprising things Newton did was show that the Earth’s rotational axis was affected by the Moon’s gravity acting on the Earth’s equatorial bulge.
- The orbit of the Moon is at an angle to Earth’s equator, so sometimes the Moon is above the equator, sometimes below.
- The Moon thus exerts an upward pull on Earth’s bulge on one side of its orbit and a downward pull on the other.
- The rotating Earth responds to this pull like a gyroscope or spinning top to a perturbing force and precesses about its axis of spin.
- Newton calculated the rate of the precession as ~50’’ per year, which is (impressively) the right value.
- However, he was using incorrect data and the result was a fortunate cancellation of errors.
- Later however, and with more accurate data, Swiss mathematician Leonard Euler proved his theory to be correct.

The Precession of the Equinoxes

- Newton's theory explains the phenomenon called the precession of the equinoxes, by which the location of the origin of the right ascension coordinate (the so-called First Point of Aries) shifts with time. This had puzzled astronomers since Hipparchus first noted the shift in the second century BC. This same phenomenon produced the shift of the 'pole star' from Thuban (in Draco) in antiquity, to Polaris (in Ursa Minor) today.
- A variant of this theory can also explain Halley's discovery of the secular acceleration of the Moon. It turns out the tides raised by the Moon on Earth provide a force that moves the Moon slowly away from the Earth, while the Earth's spin is slowed in response. The outward drift of the Moon was later confirmed by the Apollo project.

Objections to Newton’s Gravity

- It seems surprising today, but Newton’s theory of gravity met with strong objections when it appeared – some major, some trivial.
- The major objection was that it has an ‘occult’ quality of acting at a distance. The idea of a force transmitted through empty space was unacceptable. Newton didn’t like it either, but offered no explanation. He claimed that to be able to calculate the effects of gravity was enough. Not until Einstein’s 1915 theory was it understood how gravity acts.
- Newton’s theory displaced the popular ‘vortex’ theory of Descartes, which was easy to understand and well though of, though was not at all quantitative.
- Newton’s theory did not explain the peculiarities of the Moon’s motion (at least, at first). Other forces, that Newton hadn’t allowed for, were thought necessary to explain such anomalies.

Objections to Newton's Gravity

- The space between planets was thought to be filled with some kind of medium that could either help or hinder planetary motion. (It affected the motion of comet tails, for example). But this idea is largely a throwback to the vortex theory.
- None of these objections survived in the long term. It is of interest to note that those who opposed Newton's theories most strongly were outstanding mathematicians. Newton however, was a great physicist as well as a great mathematician and excelled at experimental physics. He probably had a stronger physical intuition than his critics and perhaps that was why his theories won out in the end.

Newton’s Successors - Halley

Edmond Halley (1656-1742)

- Mathematician
- Astronomer Royal
- Voyager
- Envoy for Queen Mary
- Publisher of Principia
- Discovered the periodic comet.

Newton’s Successors - Halley

- Before discussing the scientists who followed Newton, Halley deserves a special mention.
- His skills were impressive, as indicated here, and are not confined to pure science. His diplomatic skills were especially potent.
- He mapped the stars of the southern hemisphere as a young man and later mapped the Earth’s magnetic field for the Admiralty.
- One of his most important achievements was first to encourage and then keep motivating Newton to write the Principia. He then guided the book through its publication, while simultaneously placating Newton’s famously fractious personality. When the Royal Society found it could not pay for the publication, Halley paid from his own pocket.
- None of these actions were appreciated by the Royal Society, who took credit for publishing the book and were reluctant to pay for his services. This is perhaps revealing of the tension in the Royal Society between Newton and his rivals at the time. However Newton and his allies eventually came to dominate the Royal Society and later in life Halley became the Astronomer Royal.
- His most famous achievement is, of course, the discovery that what we now call ‘Halley’s Comet’ is periodic and has re-appeared throughout history. It was last seen in this neighbourhood in 1986.

Halley’s Table of Comets

Table 1: Parabolic Elements of 24 Comets (after Halley)

Date of Perihelion Passage	Ascending Node	Inclination	Motion	Perihelion Longitude	Perihelion Distance
1337 June 2 6 25	84°21′ 0″	32°11′ 0″	R	37°59′ 0″	0.40666
1472 Feb. 28 22 23	281 46 20	5 20 0	R	45 33 30	0.54273
1531 Aug. 24 21 18	49 25 0	17 56 0	R	301 39 0	0.56700*
1532 Oct. 19 22 12	80 27 0	32 36 0	D	111 7 0	0.50910
1556 April 21 20 3	175 42 0	32 6 30	D	278 50 0	0.46390
1577 Oct. 26 18 45	25 52 0	74 32 45	R	129 22 0	0.18342
1580 Nov. 28 15 0	18 57 20	64 40 0	D	109 5 50	0.59628
1585 Sept. 27 19 20	37 42 20	6 4 0	D	8 51 0	1.09358
1590 Jan. 29 3 45	165 30 40	29 40 40	R	216 54 30	0.57661
1596 July 31 19 55	312 12 30	55 12 0	R	228 16 0	0.51293
1607 Oct. 16 3 50	50 21 0	17 2 0	R	302 16 0	0.58680*
1618 Oct. 29 12 23	76 1 0	37 34 0	D	2 14 0	0.37975
1652 Nov. 2 15 40	88 10 0	79 28 0	D	28 18 40	0.84750
1661 Jan. 16 23 41	82 30 30	32 35 50	D	115 58 40	0.44851
1664 Nov. 24 11 52	81 14 0	21 18 30	R	130 41 25	1.02575
1665 April 14 5 15	228 2 0	76 5 0	R	71 54 30	0.10649
1672 Feb. 20 8 37	297 30 30	83 22 10	D	46 59 30	0.69739
1677 April 26 0 37	236 49 10	79 3 15	R	137 37 5	0.28059
1680 Dec. 8 0 6	272 2 0	60 56 0	D	262 39 30	0.00612
1682 Sept. 4 7 39	51 16 30	17 56 0	R	302 52 45	0.58328*
1683 July 3 2 50	173 23 0	83 11 0	R	85 29 30	0.56020
1684 May 29 10 16	268 15 0	65 48 40	D	238 52 0	0.96015
1686 Sept. 6 14 33	350 34 40	31 21 40	D	77 0 30	0.32500
1698 Oct. 8 16 57	267 44 15	11 46 0	R	270 51 15	0.69129

*Halley's Comet

From: Halley and His Comet by P. Lancaster-Brown.

Halley’s Table of Comets

- Halley used Newton’s method to determine the orbits of 24 historical comets for which reliable data was available.
- The calculated orbital elements are tabulated here – taken from the book by Peter Lancaster-Brown.
- Three comets underlined in this list have almost identical orbital elements. Halley therefore suggested it was the same comet re-appearing at 76 year intervals.
- He predicted the return of the comet in 1758 and it was duly sighted in December of that year. Halley himself was not there to see it; he died in 1742.

Taking Gravity Forwards from Newton

- L. Euler (1707-1783) – Cast Newton's theory into analytical form, though he initially had doubts about the inverse square law.
- A.C. Clairaut (1713-1765) – Derived an improved theory of the Moon based entirely on Newton's inverse square law.
- J-L Lagrange (1736-1813) – Reformulated Newton's mechanics and applied it to astronomy. He studied Lagrange points, perturbation theory, inequalities in planetary orbits, and the stability of the solar system.
- P-S. Laplace (1749-1825)– Applied Newton's mechanics to the solar system, he studied perturbation theory, tides and orbit determination.
- C.F. Gauss (1777-1855) – General mathematical astronomy, perturbation theory, treatment of experimental error, orbit determination.
- E.W. Brown (1866-1938) – Developed complete theory of the Moon, which was accurate to 0.001 arc second and is the basis of the modern computational model.

Newton’s Achievement in Stamps!

Newton’s Achievement in Stamps!

- Newton’s theory is celebrated in this issue of stamps in 1987. (What could be more British?) As a conclusion to this talk I’d like to point out what each stamp represents.
- The first stamp celebrates the apocryphal tale of the falling apple that inspired Newton’s first steps into the theory of gravitation. In the background is a diagram from Newton’s proof that a finite spherical mass of matter can be represented by a single mathematical point without affecting the gravitational force beyond the extent of the sphere.
- The second stamp celebrates his explanation of planetary motion and the confirmation of Kepler’s laws. It also shows the diagram of the precession of the apsides, which Newton used to explain the motion of the Moon.
- The third stamp celebrates Newton’s (uncredited) invention of the artificial satellite. A canon ball shot horizontally from the top of a mountain generally follows an elliptical orbit which normally intercepts with the Earth’s surface. But if the ball could be projected fast enough, it would miss the Earth and follow the elliptical orbit forever.

Newton’s Legacy

Taking mathematics from the beginning of the world to the time of Newton, what he has done is much the better half.

Gottfried Wilhelm Leibnitz

The End