A Note on the Motion of Star Images across a Camera Sensor

In order to describe a star's motion across the sensor of a camera we need to know a few things:

1. The rate at which stars move along the celestial equator.
2. The effect a star's declination has on its rate of movement.
3. The focal length of the telescope or camera lens.
4. The dimensions of the camera's sensor.

These will be discussed in turn below.

1/. Star motion along the celestial equator

The rate at which the stars move across the sky is determined by the rate of rotation of the Earth (ω_0) and the declination of the star (δ). The rate of rotation is set by the sidereal day, which is 23h 56 m or 86160 s. This means that

$$\omega_0 = \frac{2\pi}{86160} = 7.292462055 \times 10^{-5} \text{ radians/sec.} \tag{1}$$

We can also obtain the equivalent value in seconds of arc per second of time (a common requirement) using the formula

$$\tilde{\omega}_0 = \frac{360 \times 3600}{86160} = 15.04178273 \text{ arc-sec / sec,} \tag{2}$$

where the term 360×3600 converts 360 degrees into the corresponding number of arc seconds. The constant $\tilde{\omega}_0$ is the number of arc seconds through which a star on the celestial equator moves in one second of time. $\tilde{\omega}_0$ in equation (2) is a more memorable number than ω_0 in equation (1), since it is almost 15 arc-sec / sec.

2/. The effect of declination on the motion of stars

Stars on the celestial equator have zero declination i.e. $(\delta = 0)$. Stars not on the celestial equator, i.e. with $(\delta \neq 0)$, must move at a slower rate, as indicated in Figure 1. Stars at declinations with $\delta \neq 0$ move through the same Right Ascension angle θ, as at the celestial equator, but the distance moved in the sky differs by the factor of $\cos(\delta)$. This is because stars on the celestial equator have a radius of turn equal to the radius R of the celestial sphere (whatever magnitude is assigned to this), while the radius of turn at a declination $\delta \neq 0$, is given as $R\cos(\delta)$. Because of this difference, the angular rate of motion ω_δ at $\delta \neq 0$ is given by

$$\omega_\delta = \omega_0 \times \cos(\delta) \text{ in radians / sec,} \tag{3}$$

or

$$\widetilde{\omega}_\delta = \widetilde{\omega}_0 \times \cos(\delta) \text{ in arc-sec / sec} \tag{4}$$

It follows from (3) that ω_δ is largest when $\delta = 0$, in which case $\omega_\delta = \omega_0$, and it is zero when $\delta = 90^o$ and $\cos(\delta) = 0$.

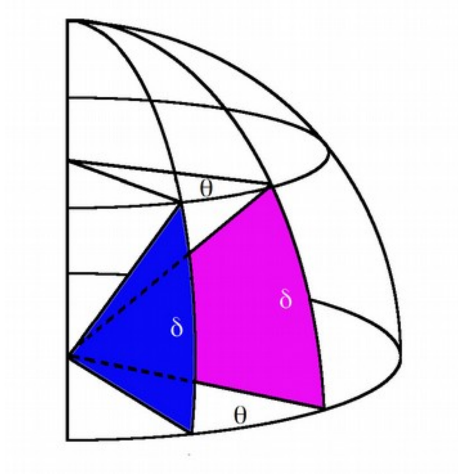

Figure 1

3/. The Effect of the focal length

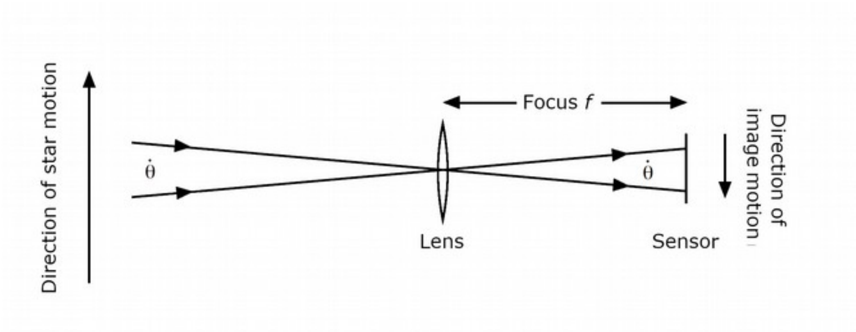

Figure 2

The effect of the telescope or camera lens of focal length f on the motion of a star's image across the sensor is shown in Figure 2. The lens inverts the direction of motion the star's image with respect to the sky, though the angular rate of turn $\dot{\theta}$ (pronounced theta-dot) is the same for the camera as in the sky (albeit in the opposite sense directionally) and so $\dot{\theta} = \omega_\delta$. The rate of movement v of the star's image across the camera's sensor is therefore given by

$$v = \omega_\delta \times f . \tag{5}$$

If the focal length is specified in millimetres (as it usually is), the formula (5) returns v in units of millimetres / second. Note that it is *not correct* to use the constant $\widetilde{\omega}_\delta$ from equation (4) for this calculation, the units of angle *must* be radians for equation (5) to hold true.

We may of course write (5) in the equivalent form

$$v = \omega_0 \times \cos\delta \times f, \tag{6}$$

which fully specifies all the required quantities on the right hand side.

4/. The dimensions of the camera sensor.

Cameras come in all shapes and sizes and so we will focus on DSLR (or Digital Single Lens Reflex) cameras, which are by far the most common cameras used in astrophotography. However, we note that the adaptation of what is written here to other types of camera is not difficult.

The sensor in a DSLR camera is descended from the original photographic film, which was most often 35 mm film with a ~3/2 aspect ratio (i.e. film width / film height = 1.5). From these specifications it follows that the active surface of the film was 29.1 mm by 19.4 mm. However, most DSLRs produced to date are somewhat smaller than this, by a factor of 0.8 or so. This means a typical DSLR sensor has the dimensions of 23.3 mm by 15.5 mm, which is equivalent to a 28 mm film. (We note here the use of millimetres to specify sizes, which corresponds with these units also specifying lens diameters and focal lengths.) A typical DSLR camera may also have a sensor composed of 6 Megapixels, which forms a 3000 x 2000 grid of square pixels with an aspect ratio of 1.5. Another common pixel count is 24 megapixels, which forms a grid of 6000 x 4000 pixels.

The first issue we tackle is the question of how long it takes for a star image to move across the camera sensor if the camera is stationary. This question is relevant to the issue of 'star trailing' which occurs when a fixed camera is used to take wide field shots. If we denote the sensor width as w, from (6) we can write the time for a star to traverse this width as

$$t = \frac{w}{v} = \frac{w}{\omega \times \cos(\delta) \times f}. \tag{7}$$

We note immediately that the only variable the astronomer has any control over is the focal length f, all other variables are defined either by nature or the camera. Setting $w=23.3$ mm (for our typical camera), $f=500$ mm (for a typical small telescope) and $\delta=0$ for fastest movement of stars, we find from (7) that $t=639$ s, which is close to 11 minutes. How long does it take to move across 1 pixel? A pixel width is 7.77×10^{-3} mm for a 6 megapixel sensor and half that for 24 megapixels. Putting these widths into (7) gives 0.2 s and 0.1 s respectively. A typical star image is of order 0.16 mm in diameter, two stars in contact on the image are therefore 0.16 mm apart, which can be taken as the minimum separation at which separate stars can be observed. Putting this width into (7) reveals that after 4.4 s, it would be possible to tell that a star image is no longer a circle. This is the order of magnitude of time for which a camera can be held fixed without star trailing becoming apparent. However, much depends on the quality of the eyesight of the viewer, and it is often quoted that something more like 8 seconds is acceptable in practice.