An Advanced Haig Camera Mount

Every astrophotographer knows what a Haig mount is. It is a simple, hand-driven device that compensates for the earth's rotation to allow photography of the night sky free of star trails. The basic idea is shown in Figure 1.

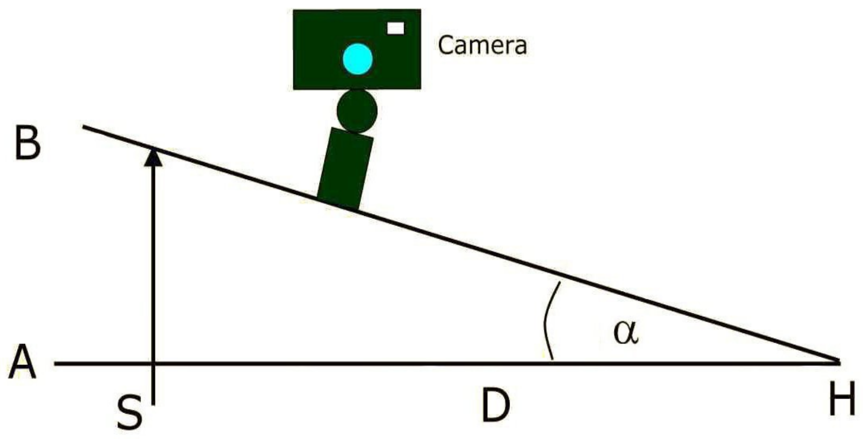

Figure 1

Two flat boards HA and HB are joined by a hinge at H. A screw S, at a distance D from the hinge, moves upwards through board HA and pushes board HB, causing it to turn through the angle α. As it does so, the camera, which is mounted to the board AB turns through the same angle. The device is set up so that the axis of the hinge points to the pole star and, with the correct rate of turning of the screw, the camera tracks with the stars. The required rate of turn for α is close to 0.25 degrees per minute and the distance D in the figure is chosen so that one turn of S per minute will produce that rate of turn in α. How D can be calculated is given in many places, including my article: *The Basic Haig Mount.*

The Haig mount is simple and effective, but it is limited to short camera exposures, perhaps a few minutes or so. This is because the device, as it is normally constructed, produces a constant rate of change in the *tangent* of the angle α rather than in the angle itself, so it cannot be accurate for more than than about 10 degrees. After a while the tracking falls behind the sky's rotation and star trails appear in the photographs. Attempts have been made to correct this defect, for example using a twin arm drive, such as the one described by Jeff Sutton in the SPA Popular Astronomy magazine for November-December 2012, which could track accurately for 100 minutes. There is however, a more simple approach, which does not seem to be well known, but which requires only a moderate change to the basic Haig design, but can extend exposure times to several hours. In short, it is to replace the screw that pushes the upper board by a cam that is shaped to compensate for the tangent effect inherent in the original design.

The principle is shown in Figure 2. HB is the upper board, as before, and HA is the lower board, slotted to allow the cam OPEC (shaded) to push through, driven at the same rate as the screw in Figure 1.

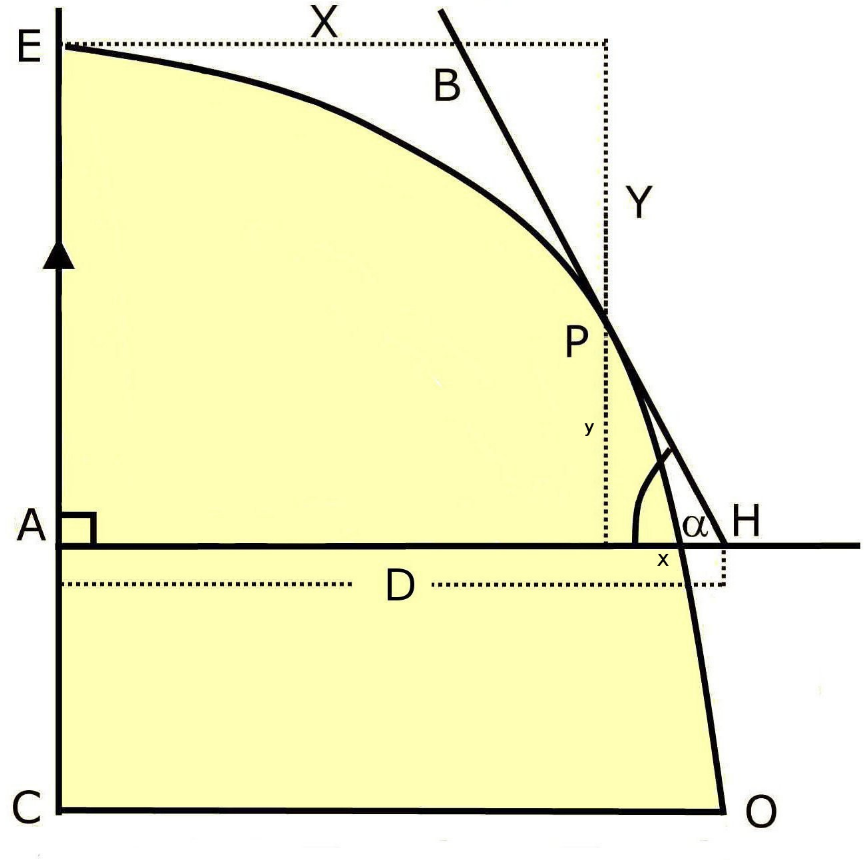

Figure 2

As the cam moves upwards it makes contact with the board HB at a moving point P (forming a tangent to the cam), and this rotates the board through the angle α. The distance from the hinge H to the point A, is D, the same distance shown Figure 1 (provided the cam is driven upwards at the same rate as the original screw). The edge of the cam EC moves perpendicular to the board HA and point E is the first part of the cam to make contact with board HB. The length of the cam (CE) is 1.57xD and the width (OC) is D. The cam can be shortened by cutting at right angles to CE, but this will reduce the range of exposure time possible. A full sized cam is capable of operating up to a practical limit of about 5 hours (6 hours is the theoretical limit, based on geometric considerations). But even a truncated form represents a huge extension of the capabilities of the basic Haig mount!

What is the shape of the cam? This can be obtained by plotting the variables X and Y (see Figure 2) on graph paper and using this to mark out a template for cutting. (Take care to use the same scale for both X and Y!) X and Y are given by the parametric equations:

$$X = D\sin^2\alpha,$$
$$Y = D\left(\alpha - \frac{1}{2}\sin 2\alpha\right). \qquad (1)$$

where α is the turning angle shown in Figure 2. Using the value of D appropriate for the drive rate of the cam, you can calculate X and Y with a calculator, for a series of values of α ranging from 0° to 90°. Alternatively you could work from the table below, in which X and Y are given for a cam with D=1.0. To use this for your own cam, just multiply the numbers appearing by your value for D.

X	Y
0.00000	0.00000
0.00760	0.00044
0.03015	0.00352
0.06699	0.01180
0.11698	0.02767
0.17861	0.05331
0.25000	0.09059
0.32899	0.14102
0.41318	0.20573
0.50000	0.28540
0.58682	0.38026
0.67101	0.49008
0.75000	0.61418
0.82139	0.75144
0.88302	0.90034
0.93301	1.05900
0.96985	1.22525
0.99240	1.39671
1.00000	1.57080

Then plot Y versus X to get your template. If you find the cam this generates seems a bit bulky, it is probably best to use a finer threaded screw for the driver, as this will shrink the device's overall size. There is no reason for the device to be huge and clunky! Take care not to confuse points O and E after you've cut your template, or the resulting device will be quite wrong!

Hopefully these brief details will allow you to make a useful Haig mount. If you are one of the many amateur astronomers who build your own stuff, I'm sure you are well capable of producing a winning device! Most care is needed in cutting the cam, as that determines the accuracy of the tracking. If you have machine tools for this, so much the better. Personal refinements could include driving the device with an electric motor. This would be so much better that driving the cam by hand for 2-3 hours - and well worth the effort!

For those interested in the derivation of the formulae used here, there follows an account of this below.

Derivation of the Advanced Haig Formulae

Consider the triangle in Figure 3 in which we imagine that the angle α is varying linearly with time (i.e. $\alpha=\omega t$, where ω is a *constant* angular velocity). We then also assume that the triangle base x is constant and ask what is the rate of change of the height h, which corresponds to a velocity v.

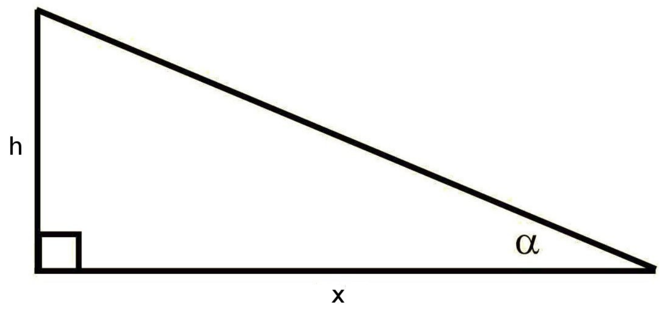

Figure 3

By simple trigonometry we have:

$$h = x \tan \alpha . \tag{2}$$

If α increases with time, while keeping x fixed, the rate of change of h is given by:

$$\dot{h} = (x\, sec^2 \alpha)\dot{\alpha}, \quad \text{i.e.} \quad v = (x\, sec^2 \alpha)\omega . \tag{3}$$

From equation (3) we can see that v is not a constant under these circumstances. Despite the fact that x and ω have fixed values, there is a dependence on α that prevents this. We can also show quite easily that v rises to *infinity* as α increases towards 90° and that its value when $\alpha = 0^o$ is the *minimum* value it can have. Denoting the minimum value as v_{min} and also noting that $x = D$ when $\alpha = 0$ (see Figure 2), allows us to obtain the following relation from equation (3).

$$v_{min} = D\omega . \tag{4}$$

If we now consider points along the hypotenuse in Figure 3 then we find, for any angle α, the vertical velocities of the points may take on any value between zero on the right hand end and some value $v > v_{min}$ at the left hand end. It follows that for any value of α there has to be a point along the hypotenuse which possesses the vertical velocity v_{min}. Using relation (4) in (3) and rearranging we can locate the point x on the base of the triangle corresponding to this velocity. This turns out to be

$$x = D \cos^2 \alpha . \tag{5}$$

In Figure 2, this equals the distance $D - X$, which on combining with (5) gives

$$X = D - D\cos^2 \alpha \quad \text{i.e.} \quad X = D \sin^2 \alpha \tag{6}$$

This is the first equation of equations (1).

From Figure 3 we can define length y of the perpendicular to the point x as

$$y = x \tan \alpha , \tag{7}$$

which on inserting the result (5) leads to

$$y = D \tan \alpha \cos^2 \alpha , \quad \text{or} \quad y = \frac{1}{2} D \sin 2\alpha . \tag{8}$$

From Figure 2 we can see that variable y is the same as the distance AE-Y where AE is the distance $v_{min} t$ which is the distance the cam moves in time t. Combining this information with result (8) and the relation (4) leads to the result

$$Y = D\left(\alpha - \frac{1}{2}\sin 2\alpha\right) . \tag{9}$$

This is the second of the equations in equations (1), which completes the derivation.

The fundamental idea behind this approach is that the cam defined by equations (1) is shaped such that the upper board HB of Figure 1 is always tangent to the cam as it slides in the upwards direction, which makes it rotate at a constant rate. The dimensions of the cam ensure this is the required sidereal rate.