The Dynamics of Dyson Shells

by

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Introduction

The Dyson Sphere was proposed by physicist Freeman Dyson[1] as a possible habitat for an advanced civilisation capable of engineering on a stellar scale. Dyson imagined the construction of a spherical shell centred on a star, on which the inhabitants could live while drawing their energy needs from the radiation of the enclosed star. If one can accept the feasibility of engineering on such a colossal scale, it is a compelling idea. However access to the total energy output of a star is not the only consideration. How, for example, could gravity be generated on the sphere, so that the inhabitants could live in an environment for which they were biologically adapted?

One possible answer is to exploit the gravity of the enclosed star, whereby the inhabitants occupying the outer surface of the shell are subject to the gravity of the star within. For this to be feasible, the shell radius must be such that the force of gravity experienced on the shell surface is equal to that on Earth. At the same time the radius needs to be sufficiently large that the intensity of the star's radiation on the inner shell surface is not great enough to damage to the shell's structure. Whether this is possible or not depends largely on the star in question. For example, for a star like the Sun it can be shown that the radius of the shell should be of order 40.67 times less than the orbital radius of the Earth if the shell surface gravity is to equal that on Earth. Unfortunately this would make the intensity of radiation on the inner shell surface 1654 times greater than the sunshine on Earth, according to the usual inverse-square law of radiation intensity. This is undoubtedly very challenging in engineering terms. On the other hand, for a white dwarf star with the same mass as the sun and 1000th the luminosity, the shell would have the same radius but experience only 1.65 times as much radiation intensity as the Earth, which sounds much more promising. In general however, it appears from these examples that the option of using the gravity of the central star may not always be available. What we require is an approach that works for all stars.

Is there some other way to create a gravitational field on a Dyson sphere? Without anticipating future discoveries in physics that may provide a more convenient option it seems there is only one answer: the sphere must rotate about its diameter. An artificial gravity would then be available in the form of the centrifugal force that arises in a rotating frame of reference. If this idea is adopted it becomes necessary to consider other forms of Dyson shell besides the sphere, since a sphere is not necessarily as ideal for gravity generation as it is for harvesting the energy output of the central star. To evaluate alternative forms of shell we must plunge into the mathematics of rotating shells and see if any of them are viable.

The Mathematics of Rotating Shells

Since a rotating shell would have a tendency to fling objects from its outer surface if the central star's gravity was not strong enough, our assumption is that it is the *inner* surface of the shell that is inhabited and we construct our mathematical model on this assumption.

Figure 1 represents a section though a general Dyson shell. S is the star about which the shell revolves and the line S-y is the axis about which it rotates. The curved line

1Possibly channelling the ideas of science fiction writer Olaf Stapleton of “Star Maker” fame!

Q-Q represents a part of the intersection of the shell with a vertical plane containing the rotation axis Sy. The shell is assumed to be concave towards S, in order to capture the star's radiation efficiently, and also symmetric with respect to inversion through the star's centre. A line Sx drawn perpendicular to Sy from S, together with Sy defines a convenient pair of Cartesian axes, with respect to which we specify the location of the point P as the coordinates x and y. For simplicity we will restrict both variables x and y to positive values i.e. the upper right quadrant of the XY plane. Extension to the other quadrants of the XY plane follows simply from the inversion symmetry of the Dyson shell.

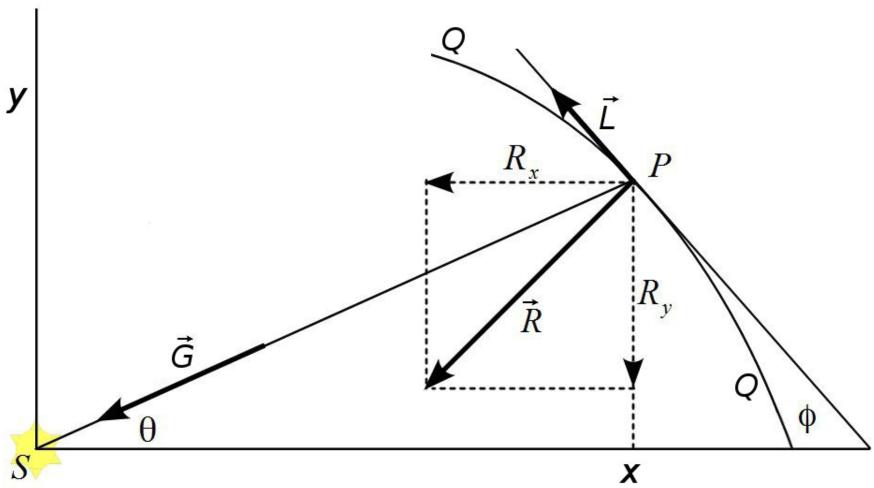

Figure 1.

In Figure 1 the angles θ and φ appear. θ is the angle made by the line SP with the X-axis. It is regarded here as a control variable for exploring how the forces change with a change in the position of P. The angle φ is the angle made by the tangent line at P and the X-axis, in the XY plane. This angle can be readily obtained if we describe the shell curve Q-Q using the formula $y=f(x)$, meaning y is some function of x, the angle φ is then given by

$$\varphi=\tan^{-1}\left(\left|\frac{dy}{dx}\right|\right), \qquad (1)$$

where the absolute value of the derivative is used. In this account we will consider a variety of forms for the function $f(x)$.

Our objective is to ensure that an object placed on the inner surface of the shell at any point P will remain there stationary despite being subject to several forces. In other words, the net sum of all the forces acting on the object must be zero. These forces are shown in the figure and are as follows.

- Firstly, the object is subject to the *gravitational attraction* $\vec{G}$ of the star at S which acts along the line PS in a direction towards the star and with a force of magnitude λ/r^2, where r is the distance from P to S and λ is the star's gravitational constant. In vector form this force is $\vec{G}=-\lambda\vec{r}/r^3$, where $\vec{r}$ is the vector from S to P.

- Secondly, due to the shell surface, the object experiences a *reaction force* $\vec{R}$ which is perpendicular to a flat plane tangent to the shell at P. The reaction force has components R_x and R_y which are resolved along the x and y axes.

- Thirdly, the object at P experiences a *lateral force* $\vec{L}$ which acts tangentially at P (i.e. is perpendicular to the reaction force) and opposes any movement along the shell surface. It is usual to think of this as a *frictional force* which acts on the object via the surface and helps to hold the object stationary. Such a force depends on the magnitude R of the reaction force and also on a friction coefficient μ, which is determined in any instance by the requirement to prevent the object sliding. But we note that it is not *necessary* to make the association with friction and indeed, it can sometimes be the case that a lateral force is needed even when the reaction force is zero. We therefore define $\vec{L}$ merely as the force necessary to stop the object sliding. It may be that friction is all that is necessary, but sometimes another contribution may be needed.

In Newtonian terms, the physics of this system can be summarise as follows: at any point P on the shell, the vectorial resultant of the lateral force $\vec{L}$, the reaction force $\vec{R}$ and the gravitational force $\vec{G}$, is the centripetal force that holds an object at P on the surface in a circular orbit around the axis of rotation Sy.

Resolving the forces into component directions x and y gives the following equations

$$R_x - L\cos\varphi - \frac{\lambda}{r^3} m x = -m\omega^2 x\,, \tag{2}$$

$$R_y + L\sin\varphi - \frac{\lambda}{r^3} m y = 0, \tag{3}$$

where m is the mass of the object and ω is the angular velocity of the shell's rotation. We also recognise the Cartesian components of the vector $\vec{L}$, which are defined as

$$L_x = -L\cos\varphi \quad \text{and} \quad L_y = L\sin\varphi. \tag{4}$$

The terms R_x, R_y, $L\cos\varphi$ and $L\sin\varphi$ are all contact forces due to the presence of the shell, so we can define a net shell force $\vec{S} = (S_x, S_y)$ with the components

$$S_x = R_x - L\cos\varphi, \tag{5}$$

$$S_y = R_y + L\sin\varphi, \tag{6}$$

The remaining forces are body forces that act on the bulk of the object. These we can write as $\vec{B} = (B_x, B_y)$ where

$$B_x = m x\left(\omega^2 - \frac{\lambda}{r^3}\right), \tag{7}$$

$$B_y = -m\,y\left(\frac{\lambda}{r^3}\right). \tag{8}$$

We can identify the body force $\vec{B}$ as the *artificial gravity* force that acts on the object. This is made clear when we reconstruct equations (2) and (3) using the definitions (5) to (8) to obtain

$$B_x + S_x = 0, \tag{9}$$

$$B_y + S_y = 0, \tag{10}$$

or in vector form:

$$\vec{B} + \vec{S} = \vec{0}. \tag{11}$$

This shows that, when the object is stationary, the (artificial) gravity $\vec{B}$ and the shell forces $\vec{S}$ cancel each other, which is the expected behaviour for an object stationary on the shell surface.

Understanding the forces $\vec{B}$ and $\vec{S}$ is essential for finding Dyson shells that are most suitable for practical habitation. Ideally we require that the artificial gravity acts perpendicularly to the shell at any point P, otherwise the local inhabitants at P would perceive that they live on an incline. In consequence moving around on the shell would be tantamount to climbing or descending a hill and free objects placed on the shell surface would slide away without the restraining force $\vec{L}$.

From equations (7) and (8) we can determine the body forces operating at any point on the shell and via the equivalence of $\vec{B}$ and $\vec{S}$ proceed to obtain the reaction force $\vec{R}$ and the lateral force $\vec{L}$ as follows.

From Figure 1 we see that

$$R_x = -R\sin\varphi \quad \text{and} \quad R_y = -R\cos\varphi, \tag{12}$$

which applies if the reaction force is more-or-less directed towards the star. So from (5) and (6) we have

$$B_x = R\sin\varphi + L\cos\varphi, \tag{13}$$

$$B_y = R\cos\varphi - L\sin\varphi, \tag{14}$$

where we have replaced the components of $\vec{S}$ with those of $\vec{B}$ as given in equations (9) to (11). Multiplying (13) by $\sin\varphi$ and (14) by $\cos\varphi$ and summing leads to

$$R = B_x\sin\varphi + B_y\cos\varphi. \tag{15}$$

The angle φ is defined by equation (1). Once R is known, components R_x and R_y can be determined from (12).

Alternatively, multiplying (13) by $\cos\varphi$ and (14) by $\sin\varphi$ and subtracting leads to

$$L = B_x \cos\varphi - B_y \sin\varphi. \tag{16}$$

Once L is known, the vector $\vec{L}$ is determined from its components, which are given in equations (4). With this result we have solved the equations of the system.

Other considerations

An additional quantity it is useful to calculate is the angle η between $\vec{R}$, which defines a direction perpendicular to the shell surface, and $\vec{S}$, the surface force vector. Through the equivalence of $\vec{S}$ to $-\vec{B}$ (the inverse of the artificial gravity) given in equation (11) this angle defines the *gravity tilt parameter* at location P. Ideally $\cos\eta = 1$, meaning gravity acts perpendicular to the floor. The tilt parameter is given by

$$\cos\eta = \frac{\vec{R}\cdot\vec{S}}{RS} \quad \text{or} \quad \cos\eta = -\frac{\vec{R}\cdot\vec{B}}{RB}. \tag{17}$$

An important operational parameter is the intensity of the star's radiation that the shell inhabitants experience. For a simple Dyson sphere of radius D, a star of luminosity Ψ at the centre subjects the inner shell of the sphere to radiation of intensity I given as

$$I = \left(\frac{\Psi}{4\pi D^2}\right). \tag{18}$$

So, if there is an agreed safe level of radiation, I_0 say, the radius of the sphere should be set as

$$D_0 = \left(\frac{\Psi}{4\pi I_0}\right)^{1/2}. \tag{19}$$

This sets the required scale of a Dyson sphere, but in the case of a different shell shape, this sets a minimum acceptable distance between the star and any point on the shell.

Another important operational consideration is the need to ensure that the artificial gravity is suitable for human beings. This means it should be as close as possible to the gravitational force experienced on the Earth, which produces an acceleration of 9.81 m/s^2. If we consider the point where the X-axis meets the shell surface in Figure 1 at a distance D_0, we can adapt equation (7) and write

$$mg = m D_0 \left(\omega^2 - \frac{\lambda}{D_0^3}\right), \tag{20}$$

Where mg represents the artificial gravity force. Rearranging (20) gives the angular rotation rate ω as

$$\omega = \pm\left(\frac{g}{D_0} + \frac{\lambda}{D_0^3}\right)^{1/2}. \tag{21}$$

A rotation in either the positive or negative sense is acceptable. This calculation defines ω for the Dyson shell as a whole.

Alternative Dyson Shells

We can now consider different shapes for Dyson shells. These we obtain by considering several different forms for the shell curve $f(x)$, all of which comply with the conditions of concavity and overall inversion symmetry with respect to the star. We present results for two kinds of star: the Sun and a white dwarf star of the same mass and with a luminosity $1/1000^{th}$ of the Sun. In both cases the equations (19) and (21) are used to determine the size scale and rotational velocity respectively.

Taking the Sun in the first instance, the normal intensity of illumination experienced on Earth, the light intensity condition (19) inevitably returns $D_0 = 1.496 \times 10^{11}$ m, which is the average Earth-Sun distance. With the given constants $\lambda = 1.3272 \times 10^{20}$ m^3 s^{-2} and g=9.81 m/s^2, this gives the result of equation (21) as $\omega = 8.1 \times 10^{-6}$ radians s^{-1} or 40° per day. (This is much faster than the rate of the Earth orbiting the Sun, which is of order 1° per day.) The Sun's gravity actually makes very little contribution to the force B in this case and neglecting it altogether, by setting $\lambda / r^3 \to 0$, gives essentially the same answer. However, the same calculations for a white dwarf star with the same mass as the Sun and with 1000^{th} of its luminosity, gives $D_0 = 4.731 \times 10^9$ m and $\omega = 5.77 \times 10^{-5}$ radians s^{-1} or 288.5° per day! The gravity of the dwarf star contributes about 38% to this result.

The results are given in the form of plots of calculated properties given as a function of the angle θ of Figure 1 (i.e. for various positions P). The properties calculated are:

- Plot D - the distance from the star to point P,
- Plot B - the scalar gravitational force B,
- Plot R - the scalar reactive force R,
- Plot L - the scalar lateral force L,
- Plot E - the gravity tilt parameter $\cos\eta$, and
- Plot I – the local radiation intensity I.

Note that plot D is given in units of the Earth-Sun distance and the forces in plots B, R and L are given in units of the gravitational force at the Earth's surface (i.e. $mg \equiv 1$). Plot I is in units of the Sun's radiation at the Earth's surface. In terms of these units the ideal values for B and I are unity and for L the ideal value is zero – meaning no lateral force is required to hold an object in place on the shell.

The Dyson Sphere

The equation of the shell curve in this case is

$$r^2 = x^2 + y^2, \tag{22}$$

where r is the radius of the sphere. The properties obtained in this case are shown in Figure 2 and Figure 3, for the model Sun and white dwarf respectively.

In Figure 2, the line labelled DI plots the distance D and the radiation intensity I for various values of the angle θ. Both of these variables hold the value 1, as expected for a spherical shell based on characteristics of the Earth-Sun system, but the other variables are less ideal.

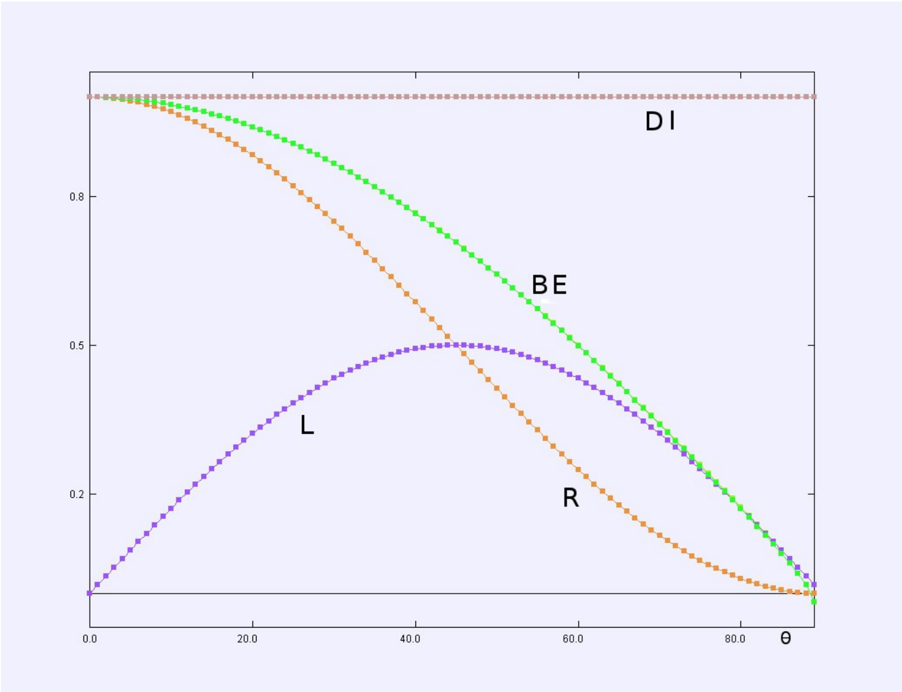

Figure 2. The Dyson sphere for the Sun

Over most of the range of θ the artificial gravity B and the gravity tilt parameter E follow similar lines, but at about $\theta = 80^o$ they separate and plot B then follows the lateral force L while line E continues down towards a negative value at $\theta \approx 89^o$. Line B is thus unfortunately obscured in the figure. However it is in fact decreasing monotonically over the range of θ towards the value λ / r^2 at $\theta = 90^o$. It can be shown that the similarity of plots B and E over most of the range is accidentally due to the choice of units for B and to the fact that the Sun's gravity makes a negligible contribution in this case. The decay of the reaction force R is a response to the weakening artificial gravity B. Since the Sun's gravity is negligible here, it decays almost as $\cos^2\theta$. Also, since the artificial gravity deviates from vertical as well as decreasing in magnitude with θ, the lateral force L required to hold objects stationary first increases to a maximum at $\theta \approx 45^o$ and falls thereafter.

From the plots in Figure 2 it is evident that not all of the interior surface of the Dyson sphere is ideally suitable for inhabitation. Earth-like conditions occur at the equator, but these become less favourable as θ increases. Movement towards the poles of the rotation axis is tantamount to climbing a hill that becomes increasingly steep. How far away from the equator it becomes unreasonable to inhabit is debatable, though there will be advantages to manufacturing in the zones of weak gravity. At the poles of the rotation axis the lateral force L, the reaction force R, and the centrifugal term $\omega^2 x$ all drop to zero, which leaves only the Sun's gravity $-\lambda y / r^3$ to govern the behaviour of objects on the shell surface. Objects placed here will inevitably fall into the Sun!

The corresponding results for the model white dwarf star are shown in Figure 3. There are some significant differences from the Sun model. Firstly, the appropriate radius for the sphere is only 0.03162 in units of the Earth-Sun distance. This is the constant value shown in the plot D.

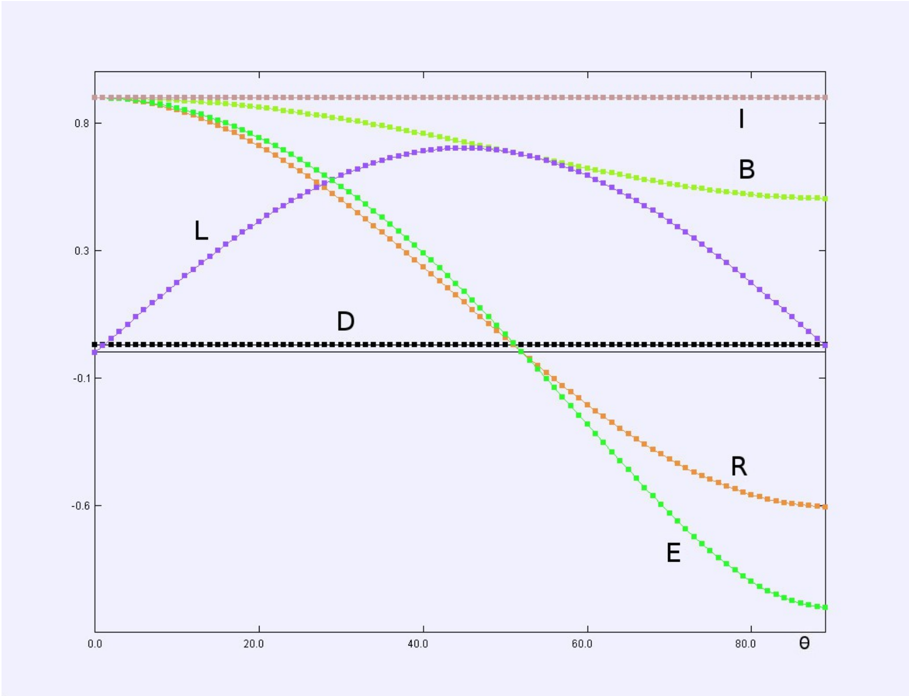

Figure 3. The Dyson sphere for a white dwarf

The radiation intensity I is however constant at the desired value 1, as expected. Unlike the previous case, the magnitude of the artificial gravity B remains large and only weakens by ~40% over the range of θ. However the direction of this force is problematic, as we shall see. The reactive force R behaves strangely, its magnitude drops to zero at $\theta \approx 52^o$, after which it becomes negative. This unexpected behaviour is matched by that of the gravity tilt parameter E. This also switches from positive to negative at $\theta \approx 52^o$. The cause of this behaviour is that at $\theta \approx 52^o$, the artificial gravity becomes tangent to the Dyson surface. For $\theta < 52^o$, it acts towards the inner surface of the shell and for $\theta > 52^o$, it acts towards the outer surface. The reaction force therefore flips direction and the sign of E follows with it. At the cross-over point the reaction force R is actually zero, though the lateral force L remains finite. In this instance the lateral force cannot be provided by friction, since the frictional force is supposedly proportional to R. Overall the behaviour of the lateral force resembles the corresponding plot in Figure 2, except that it rises to a higher maximum – indicating that a stronger lateral force is required when the true gravity is larger.

For the inhabitants of the sphere the cross-over point in this system means that there are zones on the sphere shell where habitation must switch from the inner to the outer surface and at points close to these zones the artificial gravity is ineffective. These results show that, despite providing the optimal radiation intensity on the inner surface, the sphere is not ideal for generating artificial gravity.

The Cylindrical Shell

A notable failing of the Dyson sphere is that the artificial gravity behaves erratically as the poles of the axis of rotation are approached. Since this arises from the weakening of the $\omega^2 x$ term in equation (7) it is evident that things would improve if this term could remain constant. This leads us to the idea of using a cylinder, for which the relevant equation is

$$y = x\tan\theta, \qquad \text{with} \quad x = D_0 \quad \text{always.} \tag{23}$$

The cylinder therefore spins about its longitudinal axis. In principle the cylinder could be of any length, provided the star is at its centre of gravity, but since the star's radiation intensity diminishes as the square of distance, occupation of the far extremes of the cylinder would be undesirable. This implies a practical limit on the length of the cylinder. Furthermore, the open ends of the cylinder do not capture the star's radiation. Though there is nothing to prevent the cylinder ends being 'capped' to facilitate this, the surfaces of the caps are not suitable for habitation and represent a waste of resources. Notwithstanding these disadvantages the cylinder does present some useful properties.

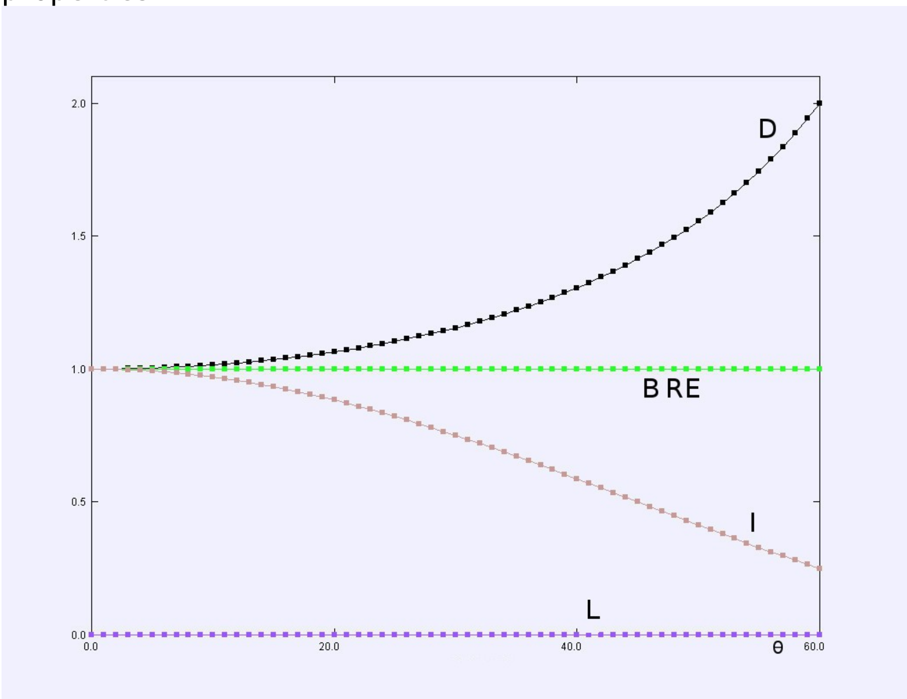

Figure 4. The Dyson cylinder for the Sun

The properties of the Dyson cylinder for the Sun are presented in Figure 4. The remarkable feature of this graph is that the artificial gravity B, reaction force R and gravity tilt parameter E maintain close approximation to their ideal value of 1. The main drawback is the increase in the distance D from the star with increasing θ. This results in a diminishing intensity of irradiation I, which falls to ¼ of the Earth's at $\theta=60^o$, corresponding to a distance of twice the Earth-Sun distance. Presumably any decrease in radiation intensity beyond this would be unacceptable. There is some merit in the characteristics of the artificial gravity in this case, but we should keep in mind the fact that the Sun's gravity has little influence here, which helps things considerably.

Results for the Dyson cylinder of a white dwarf are shown in Figure 5. As with the spherical shell, the dimension is much smaller. The distance D from the star to the surface increases with θ and the corresponding intensity of irradiation I falls exactly as before. In contrast to the previous case the artificial gravity B and reaction force R both *increase* in this case. Most significantly the artificial gravity rises to an uncomfortable 1.54 times the Earth's gravity. Similarly the lateral force L is now significantly larger than before. These effects are undoubtedly due to the increased contribution of the star's gravity. The plot of the gravity tilt parameter E stays close to the ideal value of 1 over the range of θ from $[0-60^o]$ and at worst shows a tilt of

10.5^o to the vertical at $\theta \approx 30^o$. It is because of this tilt and the strong gravity of the

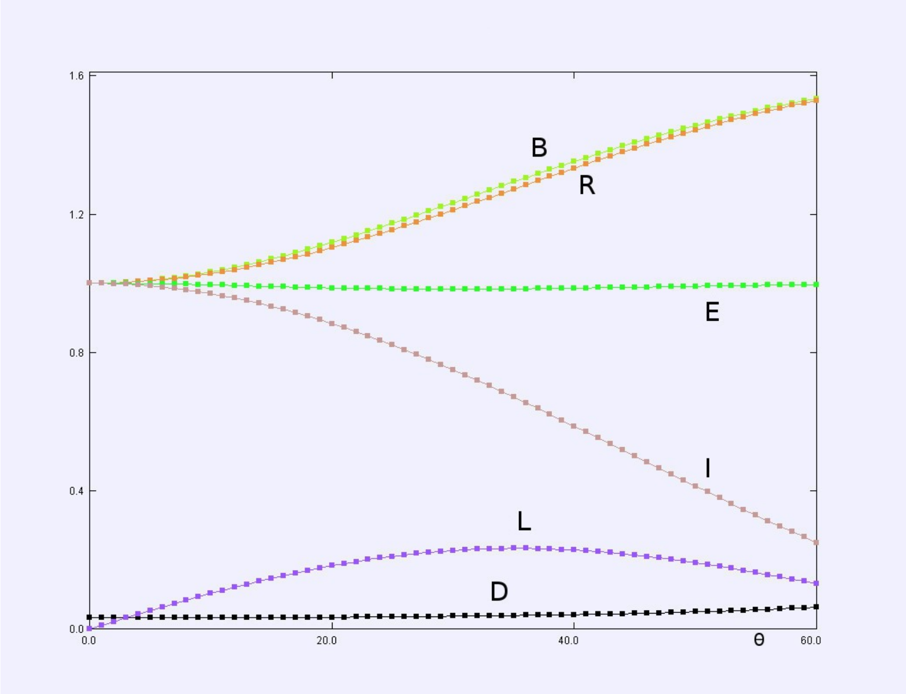

Figure 5. The Dyson cylinder for a white dwarf

star that a substantial lateral force is required to prevent sliding.

Whatever the defects of the Dyson cylinder it is attractive that there are no peculiarities in the behaviour of the artificial gravity as we move about the shell surface.

The Constant Gravity shell

It is possible to construct a Dyson shell which has a constant artificial gravity over its whole surface, thought this does not mean it is free from other defects. For example we can fix the value to mg, where g is the acceleration due to gravity on Earth. Then from equations (7) and (8) we can obtain

$$g^2 = x^2\left(\omega^2 - \frac{\lambda}{r^3}\right)^2 + y^2\left(\frac{\lambda}{r^3}\right)^2. \qquad (24)$$

This is the shell curve of the constant gravity Dyson shell. Inserting the relations

$$x = r\cos\theta \qquad \text{and} \qquad y = r\sin\theta \qquad (25)$$

allows (24) to be written as

$$r^6\omega^4\cos^2\theta - r^4 g^2 - 2r^3\lambda\omega^2\cos^2\theta + \lambda^2 = 0, \qquad (26)$$

from which r can be obtained for a given θ by Newton-Raphson iteration. Variables x and y can then be obtained from (25). In this way the shell curve can be easily obtained. We also require the derivative dy/dx, in order to obtain the angle φ from equation (1). This derivative is

$$\frac{dy}{dx}=\frac{x\{(\omega^2-\lambda/r^3)^2+3\lambda[x^2(\omega^2-\lambda/r^3)-y^2\lambda/r^3]/r^5\}}{y\{(\lambda/r^3)^2+3\lambda[x^2(\omega^2-\lambda/r^3)-y^2\lambda/r^3]/r^5\}}. \tag{27}$$

Constructing a constant gravity shell for the Sun gives the results shown in Figure 6.

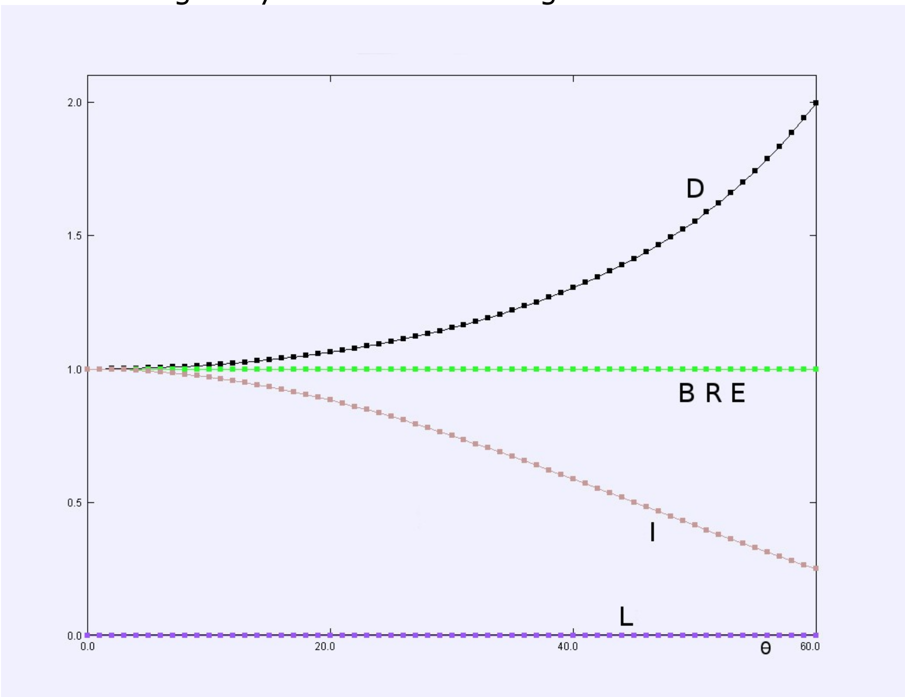

Figure 6. The constant gravity Dyson shell for the Sun

These results are remarkably like those for the Dyson cylinder (Figure 4). Indeed they cannot be distinguished on the scale drawn. The reason for this is the weakness of the Sun's gravity. Setting $\lambda/r^3 \to 0$ in equation (24) shows that we have a shell curve defined by $x=g/\omega^2$ - so x is constant. We have inadvertently recreated the cylindrical shell in this case.

However, for the white dwarf star, the star's gravity is more significant and in consequence the overall shape of the shell is roughly a cylinder which bulges in the middle. At low θ it resembles a sphere, but at large θ it is cylindrical. This gives rise to the behaviour seen in Figure 7. Once again for a white dwarf the star to surface distance D is small. Though it increases from 0.0316 at $\theta=0^o$ to 0.0441 at $\theta=60^o$, this is less than the increase seen with the cylindrical shell in Figure 5. Hence the light intensity I is 0.514 compared with 0.25 for the cylinder at $\theta=60^o$, which is a considerable advantage.

By construction the artificial gravity B holds constant at the ideal value of 1 across the range, but we note that neither the reaction force R nor the gravity tilt parameter E are constant (as they were in Figure 6). These two plots reach a minimum at $\theta=40^o$ with a value of 0.7654, which means the tilt of the artificial gravity to the local surface normal is a maximum $\eta=40^o$ at this point. This is quite inconvenient. In the same location the lateral force L required to prevent sliding is at maximum. (The equality of θ and η here appears to be accidental.) The equivalence of the plots R and E arises because the balance of forces at the surface requires that $R=-B\cos\eta$ and because we have normalised the plot B so that $B=1$. In conclusion it appears that the constant gravity shell works best when the simple cylinder is already a good

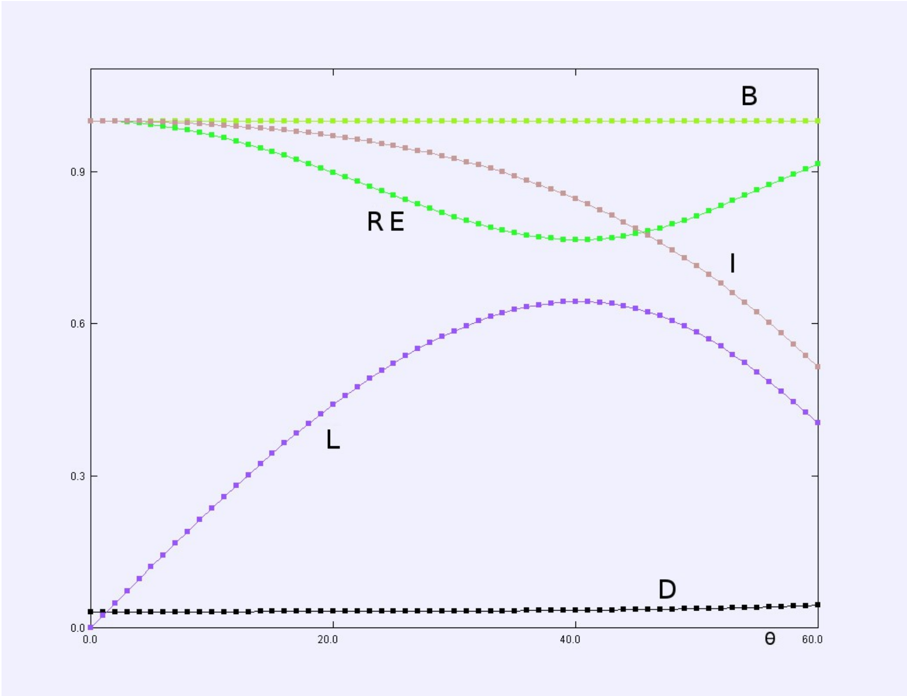

Figure 7. The constant gravity Dyson shell for a white dwarf

solution. It offers some improvement over the cylinder in the white dwarf case, but is still not ideal.

The Parabolic Shell

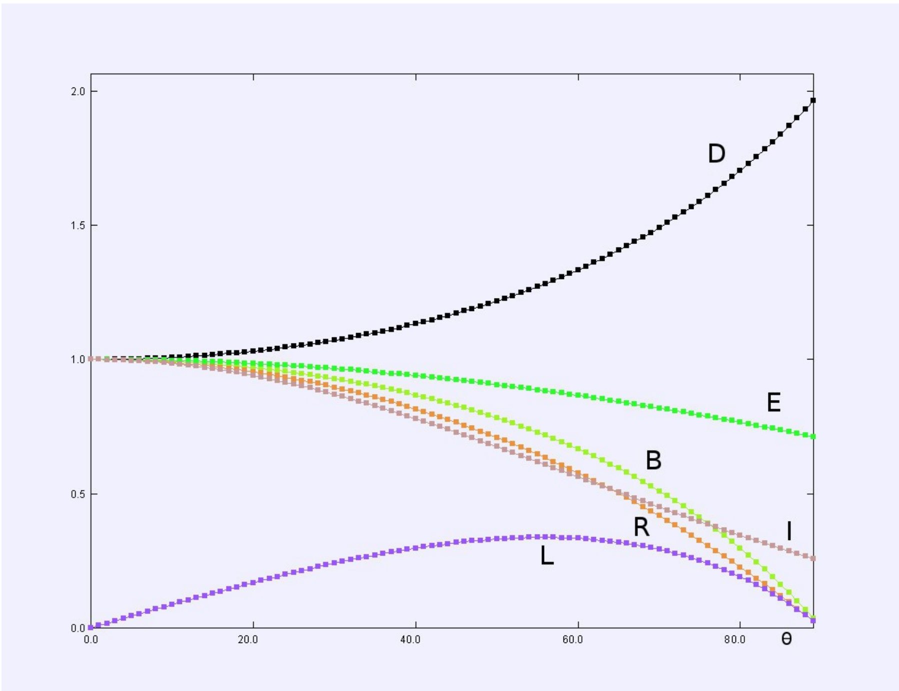

Figure 8. The Parabolic Dyson shell for the Sun

Here we consider a parabolic shell that is concave towards the star. The shell curve has the equation

$$y^2 = 4D_0(D_0 - x), \tag{28}$$

for which we note that $x = D_0$ when $y = 0$ and $y = 2D_0$ when $x = 0$. So this is a closed shell like the sphere, but it closes at the poles less rapidly with θ, which means it may not show the same disadvantages as the sphere quite so quickly.

The results for a parabolic shell surrounding the Sun are shown in Figure 8. The distance D from the star to the shell surface gradually increases with θ and the radiation intensity I inevitably decreases, so the main advantage of a sphere is lost here. However, the artificial gravity B, reaction force R and lateral force L behave in a similar manner to the sphere (Figure 2), so dynamically the system is much the same. The main noticeable difference from the sphere is that the gravity tilt parameter E changes much less, which means the gravity, though weakening, remains usable at large θ angles, as expected. However, as before, there is no stability at the poles.

Results for the white dwarf star are presented in Figure 9. Qualitatively the results

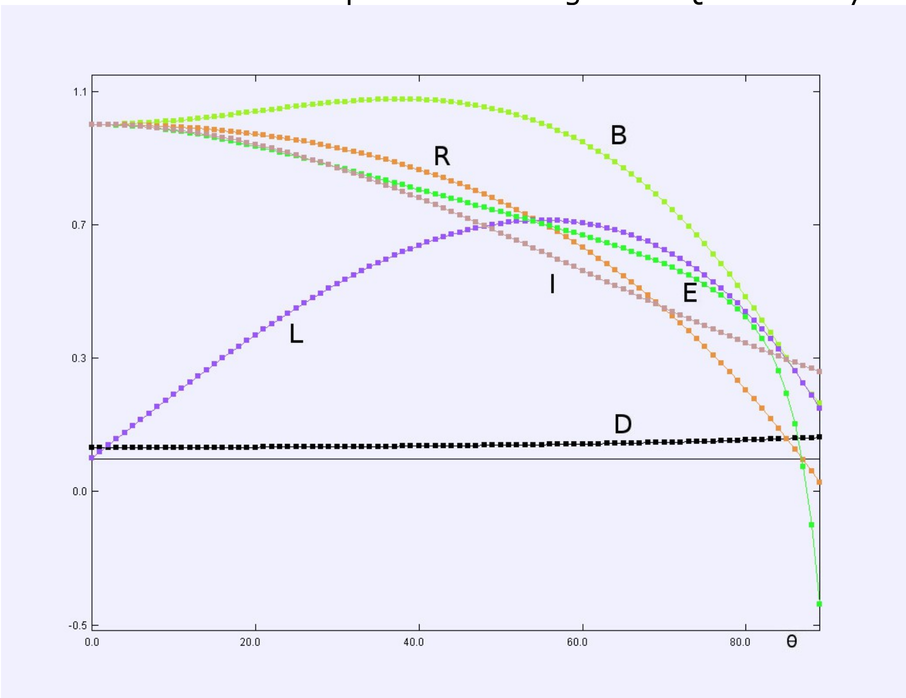

Figure 9. The parabolic Dyson shell for a white dwarf

resemble those for the spherical shell with the white dwarf (Figure 3). We again see the reaction force R and the gravity tilt parameter E eventually turn negative as θ increases, but in this case it occurs when θ is approximately 87^o, which is much later. Thus more of the shell inner surface is usable in this case. Less helpful is the increase in artificial gravity B, which is above the ideal value for much of the θ range. However the maximum increase is less than 10%. As in other cases we see the required lateral force L take on quite a large value, which shows a strong tendency for surface objects to slide. As with the Sun, the star to surface distance increases with θ and the radiation intensity I falls. However, the maximum distance D in this closed shell is $2D_0$, so the radiation never falls below 25% of the Earth value.

The Hyperbolic Shell

The final case we consider is a hyperbolic shell which has the equation

$$y^2 = 3([x - 2D_0]^2 - D_0^2), \tag{29}$$

which has the property $x = D_0$ when $y = 0$ and $y = 3D_0$ when $x = 0$ (taking the left branch of the hyperbola in both cases). This creates a closed shell like the parabolic case, except that it is elongated along the axis of rotation, compared to the parabola.

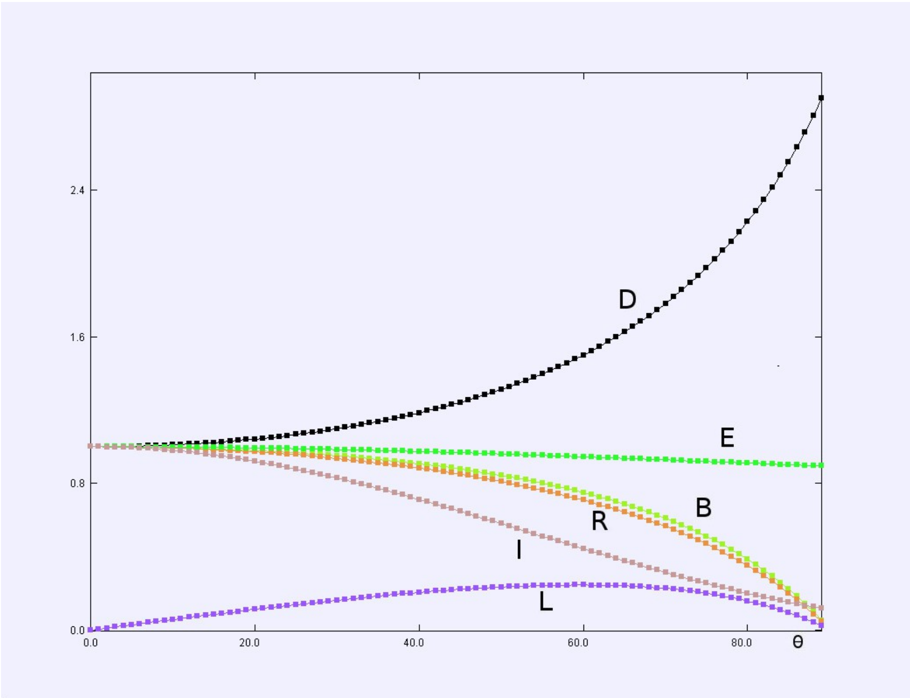

Figure 10. The Hyperbolic Dyson shell for the Sun

Results for the hyperbolic shell for the Sun are shown in Figure 10. In this case the Sun to surface distance D is rather too large at high θ and the radiation intensity I falls to an unacceptable 0.12. This implies the hyperbola is better truncated at about $\theta=76^o$, where the intensity is about 0.25. Alternatively truncating when $D=2D_0$ has the same effect. The behaviour of the artificial gravity B, reaction force R and gravity tilt parameter E is like that seen for the parabola (Figure 8) although B and R are closer in value and E remains close to the ideal value, never falling below 0.89, though the gravity becomes negligible by then. Things are better if the truncation at $\theta=76^o$ is made. The artificial gravity then falls only to about 0.5. The lateral force L rises to about 0.25 at maximum, which is better than the parabola and the sphere cases considered.

The results for the white dwarf case are shown in Figure 11. Once again the distance D increases rather too rapidly with θ and the resulting radiation intensity falls to 0.12 at the end of the range. Truncating the shell at $\theta=75^o$ stops the intensity dropping below 0.25. The artificial gravity rises to a maximum ~17% above the ideal at $\theta\approx45^o$ and drops rapidly to unsuitable values after $\theta\approx69^o$. The reaction force R behaves similarly, though the rise in value is not as great. The gravity tilt parameter drops slowly to ~80% of the ideal value by $\theta\approx80^o$ and falls very sharply soon after. The lateral force L rises to a maximum of ~0.58, which is larger than for the Sun case and shows a strong tendency for sliding to take place. All these properties suggest that truncation of the shell at $\theta=75^o$ is advisable and would leave large and usable surface for habitation.

Conclusions

It is clear from the cases considered here that constructing artificial gravity for Dyson shells by introducing rotation is possible. There are however, a number of problems arising that can only partially be alleviated by changing the shape of the shell.

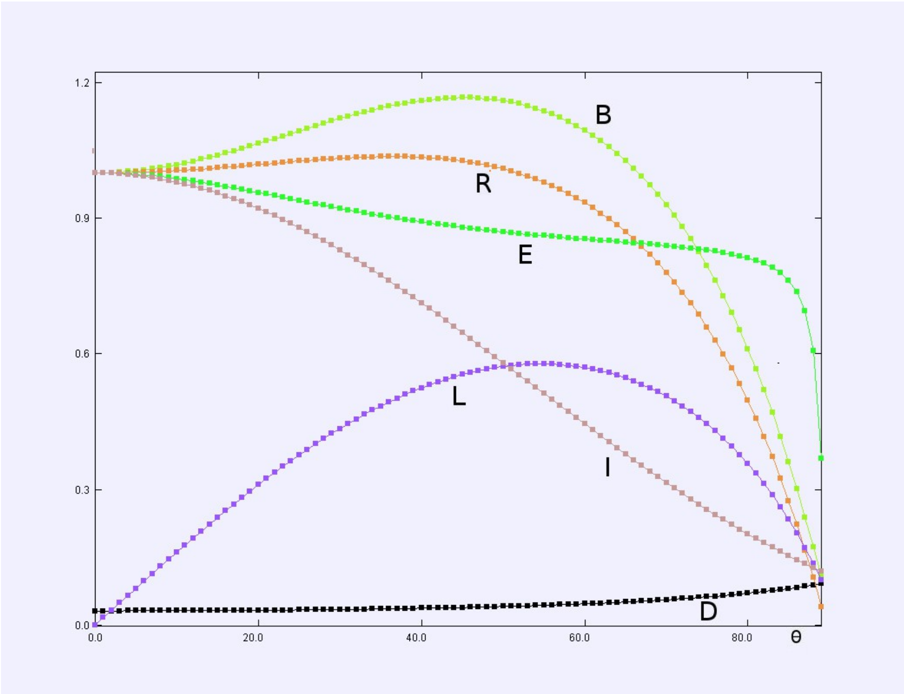

Figure 11. The Hyperbolic Dyson shell for a white dwarf

The first observation that can be made is that completely closed shells have problems at the poles of the axis of rotation. This is inevitable, since the key contribution of the rotation to the artificial gravity is the term $\omega^2 x$, which becomes negligible as $x \to 0$. It follows that, for the purposes of gravity generation, all shells have to be truncated at some value of the parameter θ to give shells that are open at the poles. This is counter to the requirement that the inner surface of the shell should also capture the radiation energy of the star, but there is nothing to prevent the shells being suitable "capped" with some, probably lightweight, structure that has the sole purpose of capturing the radiation. All of the shells described above either are, or can be, constructed in this way.

However, it must be said that not all shells are equally good at providing an artificial gravitational environment. While they all are perfectly acceptable at the equator of the rotation, they differ markedly at distances far from it. Arguably the truncated sphere is the weakest option. In the cases we have considered it is acceptable only for shells that have a small truncation angle of order $\theta \approx 20^o$. Beyond that its characteristics become difficult to deal with, particularly if the star exerts a strong gravitational influence. The truncated parabola behaves well out to $\theta \approx 40^o$ and the hyperbola to $\theta \approx 50^o$. Both may be pushed further than this if necessary. Perhaps the most surprising result is that the cylinder provides a particularly good gravitational environment and out to $\theta \approx 50^o$ it is superior to the truncated forms of the sphere, parabola and hyperbola. The constant gravity shell is also a good candidate. For Sun-like stars it has an almost identical shape and character as the cylinder, while for white dwarf stars it seems equally good in gravitational terms but has different characteristics in direction and magnitude of the gravity. Taking both gravity and available radiation intensity into account however, the constant gravity shell is best overall.