Spectroscopy and Binary Stars

Despite the large and frequently unknown distances between Earth and the stars we know a great deal about binary star systems, even if the stars cannot be optically separated and appear as a single star under magnification. This has been made possible due to the science of spectroscopy. This note provides some insight into how this is done.

In what follows, some simplifications are applied in order to bring out essential details. Most prominent is the assumption that the stars in the binary system follow circular orbits. This clearly is not true in general, but it is very common, because the tidal force between the stars tends to degrade elliptical orbits so they become more circular in time. So to begin with, it is appropriate to introduce some of the properties of circular orbits.

Circular Orbits

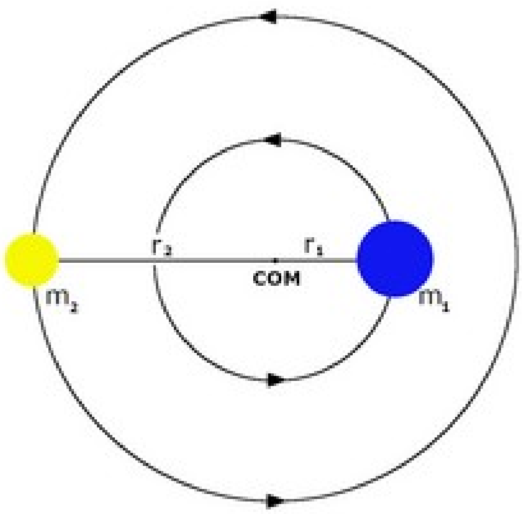

Figure 1. Binary Stars in a Circular Orbit

When we say two stars are orbiting each other in a circular orbit (Figure 1), it means that each star is following its own circular orbit[1] about their common centre of mass (COM). Furthermore a line from the centre of each star passes through the COM so the stars orbit 180 degrees apart, as Figure 1 shows. Note also that both orbits lie in the same flat plane in space.

If the masses of the two stars are m_1 and m_2 and their respective distances from the COM are r_1 and r_2, the COM is effectively defined by the relation

$$m_1 r_1 = m_2 r_2 , \qquad (1)$$

1 If the stars have the same mass, their orbits will be identical.

which we can rearrange to give

$$r_2 = \frac{m_1}{m_2} r_1 . \qquad (2)$$

The total separation r between the star centres is

$$r = r_1 + r_2 . \qquad (3)$$

Inserting equation (2) into equation (3) and rearranging gives the relation

$$r_1 = \left(\frac{m_2}{m_1 + m_2} \right) r . \qquad (4)$$

Similarly it can also be shown that

$$r_2 = \left(\frac{m_1}{m_1 + m_2} \right) r . \qquad (5)$$

If the stars follow a circular orbit then according to Newton's laws of mechanics a centripetal force $F_{centripetal}$ is required to maintain the orbit. For the star of mass m_1 this must take the form

$$F_{centripetal} = m_1 r_1 \omega^2 , \qquad (6)$$

where ω is the angular velocity of the orbital motion which is defined to be

$$\omega = 2\pi / P , \qquad (7)$$

where P is the orbital period (the time required to make one full orbit). Note that from the relation (1) equation (6) could equally well be written as

$$F_{centripetal} = m_2 r_2 \omega^2 . \qquad (8)$$

So the centripetal force is the same for both stars.

The gravitational force $F_{gravity}$ acting on both stars is given by Newton's law of gravitation as

$$F_{gravity} = \frac{G m_1 m_2}{r^2} , \qquad (9)$$

where G is the universal gravitational constant.

It is this force that supplies the centripetal force necessary to maintain the circular orbital motion and so we have

$$F_{centripetal} = F_{gravity} . \qquad (10)$$

From equations (6) and (9) it follows that for the star 1 with mass m_1

$$m_1 r_1 \omega^2 = \frac{G m_1 m_2}{r^2} . \qquad (11)$$

Using Equation (4) to substitute for r_1 in (11) gives

$$\left(\frac{m_1 m_2}{m_1+m_2}\right) r\,\omega^2 = \frac{G m_1 m_2}{r^2}. \qquad (12)$$

We can replace ω with the orbital period P using the definition given in (7). Inserting this into (12) and rearranging gives the result

$$M=(m_1+m_2)=\frac{4\pi r^3}{G P^2}, \qquad (13)$$

where M is the total mass of the binary system. In cases where the stars can be optically separated (i.e. appear as distinct stars in a telescope), the period P and the separation r can be determined by direct observation. Thus (13) can be used to determine the total mass of the binary system, which is a surprising achievement in itself.

When the stars are not optically separable we can still make progress, but we need the assistance of spectroscopy. In fact spectroscopy may be the only way in some cases to reveal that an apparently single star is in fact binary.

Spectroscopy and Binary Stars

The most important observation that can be made with a spectroscope is to reveal the presence of distinct, narrow lines at specific wavelengths in the spectrum which are characteristic of the chemical elements that constitute the stars. This is invaluable in itself but equally important is the fact that the lines invariably show a Doppler shift, which allows the determination of the speeds of stars in space relative to an observer.

The Doppler shift is an observed change in the emitted wavelength of light when the light source is receding away, or advancing towards, the observer. This phenomenon is described by the Doppler formula:

$$\frac{\delta\lambda}{\lambda_e}=\frac{\lambda_o-\lambda_e}{\lambda_e}=\frac{u}{c}, \qquad (14)$$

where λ_e is the wavelength of light emitted by the source, λ_o is the wavelength of the light received by the observer, u is the speed of the light source and c is the speed of light. The formula thus expresses the change in the wavelength $\delta\lambda$ as a function of the speed. If u is positive (i.e. the source is receding *away* from the observer), $\delta\lambda$ is also positive and so the observed wavelength λ_o is greater than the emitted wavelength λ_e. This is called *red shift*. If u is negative (i.e. the source is advancing *towards* the observer) then $\delta\lambda$ is negative and the wavelength λ_o is smaller than the emitted wavelength λ_e. This is called *blue shift*.

Applying the Doppler formula (14) to the lines seen in star spectra will sometimes reveal there are components with *different* Doppler shifts, which would indicate that more than one star is present and that they are moving with respect to each other. Furthermore, using equation (14) the speed of each component can be determined directly from the shift $\delta\lambda$, which is vital data concerning the dynamics of the stars of the binary system.

However there is a complication. Spectroscopy reveals the speeds u_1 and u_2 of the stars relative to the observer and not the orbital speeds v_1 and v_2 relative to the COM of the binary system. Furthermore even if the speed of the COM were known and subtracted, the speeds obtained would not be the orbital speeds but some component of them. The dynamics of the binary system is not therefore immediately accessible. We tackle this issue in the following session.

Determining the System Properties

From a dynamical perspective the binary system (with assumed circular orbits) is completely determined when we know the following: the period of the orbit $P;$ the separation between the stars $r;$ the stellar masses m_1 and $m_2;$ and finally the inclination of the binary orbit with respect to the line of sight from the observer to the COM. From these parameters all other dynamical properties of the system may be obtained. However, to accomplish this the system must be observed for a significant period of time - at least a period long enough to observe systematic changes in the spectral data[2]. It is then that the period P can be determined.

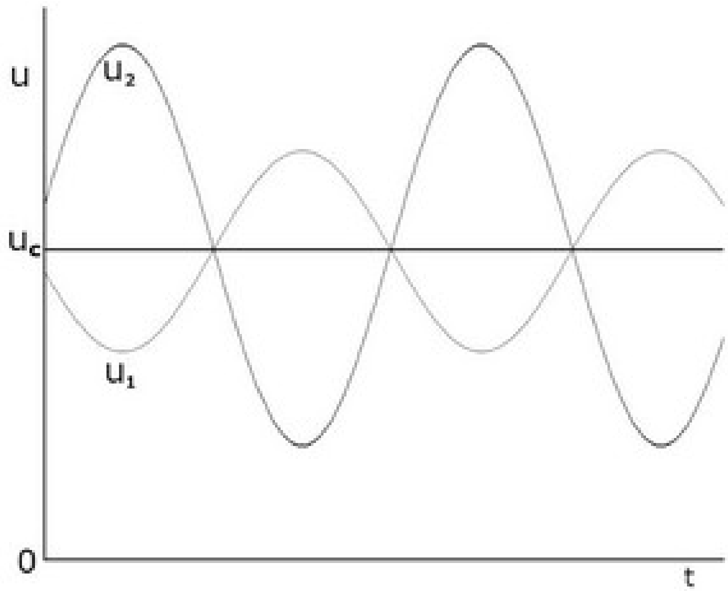

Figure 2. Speeds u_1 and u_2 plotted against time

A plot of u_1 and u_2 against time is shown in Figure 2. These are the speeds determined from the spectrum and refer to the recession or advance of the stars. The periodicity due to the circular orbit of the stars is evident. The horizontal line drawn at the speed u_c is where the two plots intersect at regular intervals of time. This is the place where the two stars have the same velocity in the direction of the observer. At such places the orbital motion is undetectable by spectroscopy because it is perpendicular to the line of sight. It follows that the speed u_c must be the speed of the centre of mass of the binary star system with respect to the observer and the interval between each intersection is half the period P of the orbit. Equivalently we may say that

2 Since these are very close binary stars, one may hope that the time period is short.

successive maxima or successive minima in the plot mark out the period of the orbit. Either way, P becomes a known quantity.

The maximum and minimum values of the plots of u_1 and u_2 minus the velocity u_c are the maximum and minimum speeds (respectively) of the stars along the direction of the observer. More generally, if we define the angle of inclination of the orbital plane with respect to the observer direction as i we can say that

$$v_1 \cos i = u_1 - u_c \quad \text{and} \quad v_2 \cos i = u_2 - u_c, \tag{15}$$

where v_1 and v_2 are the constant, circular velocities of stars 1 and 2 respectively. Rearranging (15) gives[3]

$$v_1 = (u_1 - u_c)/\cos i \quad \text{and} \quad v_2 = (u_2 - u_c)/\cos i. \tag{16}$$

Which determines the orbital speeds from the spectroscopically determined speeds u_1, u_2 and u_c. Unfortunately, we do not know the angle of inclination, but if the binary star is of the eclipsing variety, we can reasonably assume that i is near to zero, since the plane of the orbit is demonstrably near to the observer line. In which case the speeds v_1 and v_2 will be reasonably accurate.

If no eclipses occur in the period P, we can take a statistical approach and assume an average value for i based on the probability of the orientation of the orbital plane with respect to the observer line. A value of $i \sim 30^o$ will do for an estimate.

A knowledge of i, v_1 and v_2 will allow a calculation of the system mass. We know that for a circular orbit:

$$P v_1 = 2\pi r_1 \quad \text{and} \quad P v_2 = 2\pi r_2. \tag{17}$$

Using these relations in Equation (3) leads to

$$r = \frac{P}{2\pi}(v_1 + v_2) \tag{18}$$

Then substituting Equation (18) into Equation (13) and rearranging gives:

$$M = m_1 + m_2 = \frac{P(v_1 + v_2)^3}{2\pi G}. \tag{19}$$

So the total mass $M = (m_1 + m_2)$ is now a known quantity.

For a circular orbit $v = \omega r$, so we can easily show from (1) that

$$m_1 v_1 = m_2 v_2 \tag{20}$$

Using this to replace v_2 in Equation (19) and isolating m_2 on the left gives:

$$m_2^3 = \frac{P v_1^3}{2\pi G} M^2. \tag{21}$$

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3 Clearly this result is indeterminate when $i = \pm 90^o$ but then both $v_1 - v_c$ and $v_2 - v_c$ are zero and spectroscopy cannot provide an answer.

Thus m_2 is determined from known quantities on the right. The mass m_1 is then obtained by subtracting m_2 from the total mass M. We now have all the quantities we sought.

By such methods it has been possible to match the spectral types of stars in general with their masses and so underpin the theories explaining the origins and internal structures of stars.

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Sept. 2023