Superluminary Quasar Jets

by Bill Smith

Despite the fact that Einstein's theory of relativity says that no physical object can travel faster than the speed of light, there are some phenomena in astrophysics that appear to do just that. One example is the expansion of haloes associated with supernova explosions and another is the jets that fire out of the distant quasars. In both cases superluminary speeds have apparently been observed. Since both of these phenomena are associated with extreme physics it is tempting to regard these observations as evidence that Einstein's theory is breaking down under duress. However, it turns out that in neither case is it necessary to look beyond Einstein. In fact, it is not even necessary to look beyond Newton! An explanation of apparent superluminary supernova haloes is given in another article [1] , here we concentrate on quasar jets.

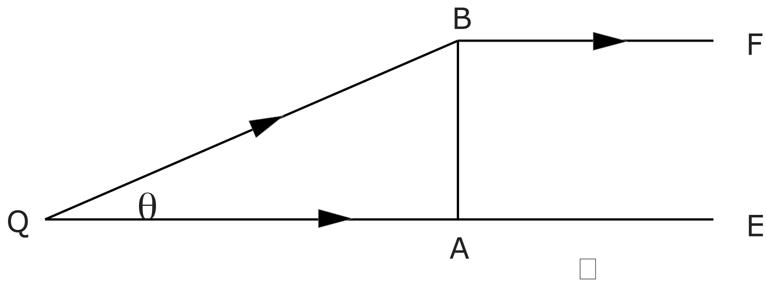

Figure 1.

The explanation for superluminary quasar jets lies in the simple geometry shown in Figure 1. Point Q represents the location of the quasar, and line Q – E a straight line drawn from the quasar to the Earth. The quasar jet is beamed out at an angle θ to the Q – E line and the point B is where the jet has reached at a time $t=t_1$, after its initial creation at Q at time t=0. The line Q – B therefore represents the direction of the quasar jet. We need only consider those circumstances in which θ lies between 0 and π radians, since this covers jets that point towards or away from the earth, and directions in between. For mathematical purposes, we construct the line B – F, which is parallel to the line Q – E and also construct line A – B by dropping a perpendicular from B to meet the Q – E line at A. By simple trigonometry we see that

$$AB = QB \sin\theta \qquad \text{and} \qquad QA = QB \cos\theta \ . \tag{1}$$

For simplicity we assume the jet travels at a constant speed, v. This is usually less than than the speed of light, c. It can be close to c, but not greater than it i.e. $v < c$ always, as required by relativity theory. We must now explain why this geometry is sufficient to show how the speed of the jet may appear to be greater than c, when observed from Earth.

The jet originates at Q at the time t=0 and light from this event travels towards Earth along the Q – E line. If this light travels with speed c, it will arrive at point A at the time t_0 given by

$$t_0 = QA / c = QB \cos\theta \ / c \ , \tag{2}$$

which follows from (1). From A the light continues towards Earth, where the start of the quasar jet is first observed.

Meanwhile the jet continues along the line Q – B and arrives at B at time t_1 given by

$$t_1 = QB / v \ , \tag{3}$$

and when it arrives at B, light emitted by the jet travels from B towards the Earth. On account of the great distance to Earth we may, with high accuracy, take this to be along a path parallel to the line Q – E i.e. along line B – F. It follows that

$$AE = BF \tag{4}$$

i.e. the distances AE and BF are the same.

Now, the light from the origin of the jet arrives at Earth at a time T_0 given by

$$T_0 = t_0 + AE / c \ , \tag{5}$$

and the light emitted by the jet at B arrives at Earth at a time T_1 given by

$$T_1 = t_1 + BF / c \ . \tag{6}$$

So from (4), (5) and (6) we have

$$T_1 - T_0 = t_1 - t_0 \ . \tag{7}$$

Equation (7) is the time, according to observations made from Earth, for the quasar jet to travel from Q to B.

If we consider the velocity vector of v, we see this can be separated into perpendicular components w and u, which are directed along the lines Q – A and A – B respectively. We can thus write

$$v^2 = w^2 + u^2 \tag{8}$$

and the question is how an observational astronomer may determine the speeds w and u and so calculate the jet velocity v.

The component w is easily obtained. It can be directly determined from the Doppler shift in the spectrum of the jet. This is a blue shift when the angle $\theta < \pi/2$ and a red shift when $\theta > \pi/2$. Such determinations indicate that the observed speed w is never greater than c. So any possibility that v may turn out larger than c must be due to the observed value of u.

The component u however, is obtained by a less direct method. It has to be obtained from an estimate of the distance AB in Figure 1, as determined from Earth, and the time interval $T_1 - T_0$ required for the jet to cross this distance. If the distance, D, from Earth to the quasar is known, the distance AB can be obtained from the angular length, φ, of the jet against the background stars. This is expressed as

$$AB = D\,\varphi\ , \tag{9}$$

and it must be admitted that while φ can be measured with some accuracy, D is usually a rough estimate. This however, is not the sole root of our difficulties.

From Earth the speed, u, of the jet perpendicular to the line Q - E can now be estimated from the relation

$$u = AB\ /\ (T1 - T0) = AB\ /\ (t_1 - t_0)\ . \tag{10}$$

Using equations (1), (2) and (3), this may be written as

$$u = QB \sin\theta\ /\ (\ QB\ /\ v - \ QB \cos\theta\ /\ c) \tag{11}$$

which can be tidied up to

$$u = \ c\,\beta\ \sin\theta\ /\ (1 - \beta \cos\theta) \tag{12}$$

in which we have defined the ratio (familiar to relativity theory)

$$\beta = v\ /\ c\ . \tag{13}$$

Since we hold that relativity theory is correct, β must be less then 1 in all circumstances.

From equation (12), we can see that u could exceed c if ever the following circumstance arises:

$$\beta\ \sin\theta\ > 1 - \beta\cos\theta \tag{14}$$

or, by rearrangement

$$\beta\ (\sin\theta\ + \ \cos\theta) > 1. \quad \text{or} \quad \beta\ 2^{1/2}\cos(\theta - \pi/4)> 1. \tag{15}$$

Equations (15) can be used to find values of β returning a superluminary value of u for a given θ, whenever that is possible. Only values of θ in the range $[0, \pi/2)$ can guarantee this, for which aacceptable values of β are in the range $(2^{-1/2}\ , 1]$. These speeds are not physically unreasonable and indicate that u can indeed exceed light speed.

What is the maximum possible value of u? We can obtain this by differentiating (12) with respect to θ, which gives

$$du\ /\ d\theta = c\ (\ \beta\cos\theta\ /[1 - \beta\cos\theta] - \beta^2 \sin^2\theta\ /\ [1- \beta\cos\theta]^2\ ). \tag{16}$$

Since this derivative is zero at maximum u, we can easily tidy up (16) into the form

$$\cos\theta\ - \ \beta\cos^2\theta\ - \beta\sin^2\theta = 0\ , \qquad \text{or} \qquad \cos\theta\ - \beta\ = 0\ , \tag{17}$$

from which it easily follows that $\beta = \cos\theta$ for maximum u. This allows us to write

(12) as

$$u_{max} = c\,\beta\,(1 - \beta^2)^{1/2} / (1 - \beta^2) = c\,\beta / (1 - \beta^2)^{1/2}\,. \qquad (18)$$

Clearly, from (18) u_{max} tends to infinity(!) as β tends to 1 (or equivalently as θ tends to 0). More prosaically, u_{max} will exceed c whenever

$$\beta > (1 - \beta^2)^{1/2} \quad \text{or} \quad \beta > 2^{-1/2} \qquad \text{i.e.} \quad v > 2^{-1/2}\,c\;. \qquad (19)$$

We conclude that it is indeed possible to observe quasar jets that have apparently superluminary speeds, but these do not contravene relativity theory.

[1] “Superluminary Haloes?”, by Bill Smith 2014.