The Vernier Scale

Most scientists and engineers know about the Vernier scale, an ingenious device for maximizing the accuracy of a ruler measurement. More specifically, if the smallest division marked on a ruler is say, a millimetre, then the accuracy of a measurement using the ruler is formally $\pm 1\ mm$. However, for such a ruler the Vernier scale enables an accuracy of $\pm 0.1\ mm$ which is better by a whole order of magnitude! How is this possible?

The Vernier scale is actually a simple device. It is a small ruler that has a measurement scale that is $9/10$ that of the ruler it is used in conjunction with, when a measurement is made. To see how this works, we first look at some properties of the combination of a ruler and the Vernier scale.

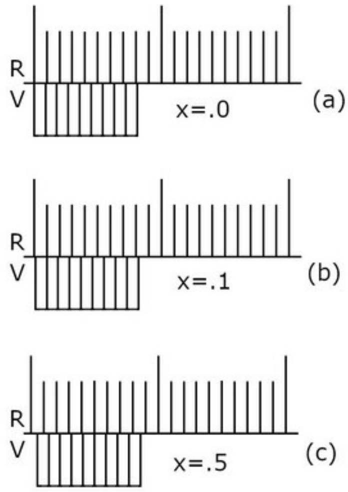

Figure 1. Properties of a ruler and Vernier scale combination

In diagram (a) of Figure 1 we see a ruler (R) laid out in the upper part and the smaller Vernier scale (V) laid out below. The tick marks on the ruler specify the smallest interval of measurement for the ruler, which is the distance from one tick mark to the next. (We shall refer to this interval as the *base interval*.) These intervals are marked out in groups of ten, which would typically correspond to one centimetre[1], which is marked off in Figure 1 by the larger tick marks.

The smaller Vernier scale marks out just 9 intervals between 10 tick marks and each interval is 9/10 of the base interval. So when the left hand edge of the Vernier scale is coincident with the left hand edge of the ruler (i.e. the *zeroth* tick marks of each scale are lined up together, as in Figure 1 (a)) it can be seen that none of the remaining tick marks of the Vernier scale are coincident with any on the ruler[2]. The interesting thing however, is that the first tick mark right of the zeroth mark on the Vernier scale falls short of coincidence with the corresponding first mark on the ruler by 1/10 of the base interval, while the second tick mark falls short by 2/10 of the base interval, the third by 3/10 of the base interval, and so on. A clear pattern thus emerges.

This means that the first tick mark on the Vernier scale can be brought into coincidence with the first tick mark on the ruler by shifting the Vernier scale to the

1 One could equally speak of inches, or some other unit, instead of centimetres.

2 If however the Vernier scale marked out 10 intervals, the 11th tick mark would line up with the 10th of the ruler, as it must, but this extra interval is unnecessary for the Vernier scale.

right by 1/10 of a base interval. This is indicated in diagram (b) in Figure 1, where a shift x, where $x=0.1$ (in units of base intervals) makes the two first tick marks coincident. Likewise, coincidence between the second tick mark of the Vernier scale with the second tick mark of the ruler can be accomplished by a right shift of 2/10 of a base interval i.e. $(x=0.2)$ from the original coincidence of the zeroth marks. And so on up to $(x=0.9)$ which brings the corresponding ninth tick marks into coincidence. Diagram (c) of Figure 1 shows the result of a shift $x=0.5$, which makes the fifth tick marks on both scales coincident.

The important point we learn from these observations is that when the n^{th} tick mark of the Vernier scale coincides with n^{th} tick mark on the ruler, we know that the zeroth mark of the Vernier scale is n/10 of a base interval (or 0.n base intervals) to the right of the zeroth tick mark on the ruler.

However, there is nothing special about the zeroth tick mark on the ruler. Any location on the ruler can serve as the starting position of the Vernier scale. In such a case we can say that the coincidence of the n^{th} tick mark on the Vernier scale with *any* tick mark on the ruler indicates that the left end of the Vernier scale is 0.n base intervals to the right of a specific tick mark on the ruler. That specific tick mark is the first to the left of the start of the Vernier scale.

We can now use this observation to improve the measure of an object’s length. If the object is first laid against the ruler in the usual way for measuring length, we will find that the right hand end of the object will usually lie between two tick marks on the ruler – indicating that the object’s length is not an exact number of base intervals, but an integer number of base intervals plus some fraction of a base interval. The fractional quantity can be obtained by placing the left end of the Vernier scale against the right end of the object and locating the Vernier tick mark that is coincident with any tick mark on the ruler. If that coincidence occurs for the n^{th} tick mark, we know that the fraction we seek must be 0.n base intervals. In this way the Vernier scale enables an increase in accuracy beyond a simple integer number of base intervals.

Lastly, what do we do if no tick marks line up? This is clearly a possibility since most objects will not be exactly some multiple of 0.1 base intervals. The resolution of this is simple. It is only necessary to identify the pair of tick marks that line up *most closely* to coincidence. The Vernier tick mark for which this is the case gives the maximum attainable accuracy for the measurement.

This is how the Vernier scale works!

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August 2023