The Real Warp Speed

by Bill Smith

Many readers will know the “Star Trek” series that has appeared at intervals on our televisions and in our cinemas since the 1960's. That being the case, the term “warp speed” will be a familiar one. I am not sure what warp speed translates into in conventional units of speed, but I had always assumed it meant a speed that was some multiple of the speed of light. This seems an essential idea, how else can a star ship scud from one stellar civilisation to another between commercial breaks? Unfortunately, as far as we know, super-luminary speeds are impossible. This is not because the technology required does not yet exist, but because the laws of physics prevent it. So in fact it will *never* be possible. Who says so? Why, none other than Albert Einstein, whose theory of relativity is based on this very impossibility. As we know, his theory is at the very heart of of modern physics, so if he is wrong we are in big trouble. So can the idea of warp speed be discarded as artistic licence or convenient fiction? Can we kiss goodbye to the idea of human colonies in far away stellar systems? It would seem so, but let's not be too hasty. Einstein was no kill-joy, so let us look at the issue more closely, starting with some aspects of his 1905 scientific paper – the puzzlingly entitled “On the Electrodynamics of Moving Bodies”, which is better known as the Theory of Special Relativity.

Special Relativity has a reputation for being difficult, largely because many of its conclusions are counter intuitive. But the theory is actually based on simple, believable principles which happen to lead to unexpected consequences. Fortunately Einstein was bold enough to hold fast to these principles, despite the all the apparent nonsense that followed, to make one of the most spectacular breakthroughs in the history of science. In essence he said two things: the laws of physics are the same for everybody, no matter what their state of motion, and the speed of light is the same for all. The first of these is uncontroversial; laws are laws and are expected always to hold true. The second however, defies common sense: how can light race away from me at the same speed, no matter how fast I chase after it? It seems crazy, but in fact the theory of electromagnetic radiation (which means light) has always implied this, but nobody until Einstein realised its significance. In consequence the independent and unchanging entities we call space and time had to be reinvented as the single, flexible entity we now call space-time.

By definition, in space-time faster than light travel is prohibited, but relativity theory also says space-time is flexible and that gives us other options, which we shall now explore. The important thing to point out is that, according to the theory, someone travelling at a constant speed with respect to someone else who is stationary measures time and space differently. It transpires that, from a stationary perspective, we would say that for the traveller time runs slower than for us and that, in the direction of his travel, for distances we measure, his measures are shorter. He of course knows nothing about this, as far as he is concerned his clocks all run at the correct rate and his measuring rulers are the correct length. But if he held these objects up for us to see as he passed, we would see his clock ticking more slowly than ours and his ruler looking suspiciously short. Ironically he would say exactly the same things about us, since from his perspective, he is stationary, while we are moving at a constant speed in a backwards direction! In relativity theory, these apparently contradictory observations are all perfectly normal. The truth is, it is our "common sense" that is at fault.

All this being so, there is an important observation we can make. Suppose that, in our stationary world, we have a straight road of length L kilometres. Someone travelling along this road at a constant speed v would determine that the road has a length, L', which is shorter than L. We can encapsulate this difference in a factor, γ, which is called the *relativity factor* and is the ratio of L to L' :

$$L/L'=\gamma .$$

Since L is greater than L', the factor γ is greater than 1.

Also, if we timed his journey along the road as T hours, he would report that it took T' hours, with T' being less than T, again by the same factor γ, so that :

$$T/T'=\gamma .$$

What is this factor γ and why does relativity say that the traveller measures times and distances differently from a stationary bystander? This surely contradicts common sense! Relativity theory calculates the factor γ as

$$\gamma=1/\sqrt{1-\frac{v^2}{c^2}},$$

which is a rather awkward formula, but you need only note that it depends only on the speed of the traveller, v, and the speed of light, c, which surely *everyone* knows is 299.78 million metres per second. Since c is such a very large speed and much larger than everyday speeds, this formula returns a value of γ very close to 1. Which means that, in ordinary circumstances, the times and distances measured by the traveller and by stationary bystanders are practically the same. This accords with everyday "common sense" and explains why we regard it as such. However, when the speed v is close to the speed of light γ becomes very large and the differences between stationary measurements and those made by the traveller are huge. (Note incidentally, that despite the differences in measured times and distances, both we and the traveller measure the speed v to be the same, since in fact $v=L/T$, which means $v=(\gamma L')/(\gamma t')$, so that $v=L'/T'$, as required.)

Now, how does all this help interstellar travel? Let us suppose we wish to travel from the neighbourhood of the Sun, to the star Deneb, which is roughly 3,000 light years away. This distance implies that, even at the speed of light, it is going to take 3,000 years to get there! But we are not looking at things properly, which is to say, relativistically. Yes, from the perspective of an inhabitant of the Solar System, the journey time could be no less than 3,000 years. But from the perspective of someone on board a star ship, relativity says that things would be different.

Since travel at the speed of light is impossible, suppose the star ship could travel at a speed $v=0.9999999c$, which is 99.99999% of the speed of light, the journey time as measured from the Solar System, T, would then be close enough to 3,000 years. We can calculate the factor γ, for this speed which gives us a value $\gamma=2236.07$. We can use this to calculate T', the journey time as determined on board the star ship from the formula $T'=T/\gamma$, which gives $T'=3{,}000/2236.07$, or 1.34 years! This is considerably better than 3,000 years and gives us hope that interstellar travel may be a practical possibility after all! Incidentally, we can do a similar calculation for distance and the result is that the distance to Deneb, according to those aboard the star ship, is 1.34 light years. The universe therefore looks smaller from the star ship, which is why, even though it is travelling at sub-light speed, it completes the journey in so short a time.

This all looks good but there are some caveats. Firstly, it will be apparent that shorter journey times aboard the star ship do not mean shorter times back in the Solar System. The calculated 3,000 years remains at 3,000 years, so however short the travel time may be aboard ship, there is no meaningful prospect of returning home. Civilisations rise and fall in less than 3,000 years, so nothing that the would be traveller holds dear is going to be there should he return. Secondly, relativity also reveals that gigantic energy resources are required to accelerate the star ship to the required speed and it may be beyond any future technology to provide it. Finally there is the human factor: passengers cannot endure acceleration rates much above 1 g (equivalent to Earth's gravitation) for any extended period. This places severe restrictions on the acceleration that can be used and if near-light speed cannot be accomplished quickly, the potential time gains from relativity may be lost.

These issues aside, it could be that interstellar travel may not be the non-starter it first appears. Which brings us back to the question: how are we to define warp speed, given that faster than light travel is impossible. If we consider this question from the point of view of those aboard the star ship, there appears to be only one option. Based on what we know about relativity theory I propose that we define the warp speed, w, as

$$w=\gamma v,$$

where γ is the relativity factor described above and v is the real speed of the star ship relative to its destination. This speed can be determined by taking the spectrum of the destination star and looking at the “blue shift” in the spectrum lines. This allows v to be calculated exactly and, of course, v (which will always be less than c) can then be used to calculate the relativity factor γ.

Now, why is w a good definition of warp speed? This comes down to what is important to the people aboard the star ship and that is how long the journey will take. Consider their point of view. They have their own clock, by which they will reckon their journey time. They know their speed, v, from spectroscopic measurements and, they will know the distance, L', to their destination (for example, from the apparent luminosity of the star as determined from the moving ship). With this information they can calculate the journey time as $T'=L'/v$. This is simple enough, but suppose they want to halve their journey time, what speed should they use then? It is not as obvious as doubling the speed v, because in relativity, changing v also changes the measure of distance and time, so things are not so simple. Twice v might even be a speed that is bigger than the speed of light, which we know is impossible.

It is here that warp speed comes to the rescue. If we use the warp speed w instead of the real speed v, it can be shown that doubling this does indeed result in the journey time being halved. The procedure is first to calculate w from the formula $w=\gamma v$, double the value, and then convert this back to a real speed. If you recalculate the journey time, you will find that it is half of the previous value. The calculations form the basis for a more general procedure for decreasing the travel time by any desired factor, α, which is:

- Obtain the current real speed v_1.
- Calculate the warp speed: $w_1=v_1/\sqrt{1-v_1^2/c^2}$.
- Increase w_1 by the required factor: $w_2=\alpha w_1$.
- Calculate new real speed: $v_2=w_2/\sqrt{1+w_2^2/c^2}$.
- The new journey time will be: $T'_2=T'_1/\alpha$.

An interesting aspect of the warp speed is that it can be many times larger than the speed of light and it is surprising that the calculations above never give a final real speed larger than the speed of light. This can easily be proved mathematically, but it can also be seen in the following table where w is presented against the corresponding real speed v. (Both speeds are presented as multiples of the speed of light.)

v/c	w/c
0.242535625	0.25
0.447213595	0.5
0.707106781	1
0.894427191	2
0.948683298	3
0.970142500	4
0.980580675	5
0.986393923	6
0.989949493	7
0.992277876	8

0.993883734	9
0.995037190	10
0.999950003	100
0.999999500	1000
0.999999995	10000
$v=1-1/(2w^2)$	> 10000
1	∞

Along with the warp speed it is useful also to define a *warp distance,* D, which is expressed as $D=\gamma D'$, where D' is any distance measured aboard the star ship. The advantage of using the warp distance is that it has a simple relationship to the "on board" time, T', and the warp speed, w, which is $D=wT'$. This simple relationship resembles the one we are most familiar with in ordinary circumstances: the distance travelled is the speed times the time. Of course this equation doesn't tell us anything more than the equivalent form $D'=vT'$, which uses the real speed and the measured distance, but it has nicer properties. For example doubling the warp speed w doubles the warp distance D that is travelled in a fixed time T'. This is not the case for the alternative form. Also D is independent of w, meaning you can change one without affecting the other, but if you change v, then you are forced by physics to change D' also, because it depends on γ. Thus the variables D and w are much more convenient variables to work with than D' and v, even though the latter are related to real measurements. Mathematically D and w represent a linearization of the effects of relativity and can be used with consistency. Once the value of v has been obtained by observation, the corresponding value of γ can be used immediately to generate variables w and D, which together with the on board time T', provide all the information necessary to describe the progression of the journey. In fact people on board may simply regard their journey distance as D and their speed as w, since it accords with their "common sense", and let the on board computer handle all the relativistic stuff!